Simplifying Expressions Involving Exponents

1.10 SKILL BUILDER

Quite often, you may have to apply several laws of exponents to simplify numerical and algebraic expressions involving exponents.

Rule	Algebraic Example	Description
product	$2^3 \times 2^4 = 2^7$	$a^m \times a^n = a^{m+n}$
quotient	$5^6 \div 5^2 = 5^4$	$a^m \div a^n = a^{m-n}$, $a \neq 0$, $m > n$
power of a power	$(3^3)^2 = 3^6$	$(a^m)^n = a^{m \times n}$
power of a product	$(2 \times 3)^4 = 2^4 \times 3^4$	$(xy)^m = x^m y^m$
power of a quotient	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $y \neq 0$
zero as an exponent	$4^0 = 1$	$a^0 = 1$
negative exponents	$6^{-2} = \frac{1}{6^2}$	$a^{-n} = \frac{1}{a^n}$, $a \neq 0$
rational exponents	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$	$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

The following examples illustrate how to simplify numerical expressions using these laws of exponents.

Example 1

Use the laws of exponents to simplify each expression.

(a) $(4^2 \times 9^4)^{\frac{1}{2}}$

(b) $\dfrac{5^{\frac{1}{4}}}{5^{\frac{1}{3}} \times 5^{\frac{3}{4}}}$

Solution

(a) $(4^2 \times 9^4)^{\frac{1}{2}}$ — Apply the power of a product law.

$= (4^2)^{\frac{1}{2}} \times (9^4)^{\frac{1}{2}}$ — Apply the power of a power law.

$= 4^1 \times 9^2$ — Evaluate 9^2.

$= 4 \times 81$ — Evaluate the product.

$= 324$

(b) $\dfrac{5^{\frac{1}{4}}}{5^{\frac{1}{3}} \times 5^{\frac{3}{4}}}$ Apply the product law.

$= \dfrac{5^{\frac{1}{4}}}{5^{\frac{1}{3}+\frac{3}{4}}}$ Apply the quotient law.

$= 5^{\frac{1}{4}-\left(\frac{1}{3}+\frac{3}{4}\right)}$ Determine a common denominator.

$= 5^{\frac{3}{12}-\left(\frac{4}{12}+\frac{9}{12}\right)}$ Simplify.

$= 5^{\frac{3}{12}-\frac{13}{12}}$ Evaluate.

$= 5^{-\frac{10}{12}}$

$= 5^{-\frac{5}{6}}$ Express using positive exponents.

$= \dfrac{1}{5^{\frac{5}{6}}}$

$= \dfrac{1}{\sqrt[6]{5^5}}$ Simplify.

$= \dfrac{1}{\sqrt[6]{3125}}$

You can simplify algebraic expressions using the laws of exponents.

Example 2

Simplify each expression.

(a) $\left(\dfrac{\sqrt[4]{x^7}}{\sqrt{x}}\right)^8$ (b) $\dfrac{(x^{2n+1})(x^{3n-1})}{(x^{2n})^3}$

Solution

(a) $\left(\dfrac{\sqrt[4]{x^7}}{\sqrt{x}}\right)^8$ Rewrite with rational exponents.

$= \left(\dfrac{x^{\frac{7}{4}}}{x^{\frac{1}{2}}}\right)^8$ Apply the quotient law.

$= \left(x^{\frac{7}{4}-\frac{1}{2}}\right)^8$ Determine a common denominator.

$= \left(x^{\frac{7}{4}-\frac{2}{4}}\right)^8$ Simplify.

$= \left(x^{\frac{5}{4}}\right)^8$ Apply the power of a power law.

$= x^{\frac{40}{4}}$ Simplify.

$= x^{10}$

(b) $\dfrac{(x^{2n+1})(x^{3n-1})}{(x^{2n})^3}$ — Apply the product and power of a power laws.

$= \dfrac{x^{2n+1+3n-1}}{x^{2n \times 3}}$ — Simplify.

$= \dfrac{x^{5n}}{x^{6n}}$ — Apply the quotient law.

$= x^{5n-6n}$ — Simplify.

$= x^{-n}$ — Express using positive exponents.

$= \dfrac{1}{x^n}$

Key Ideas

- When simplifying expressions involving exponents (positive, negative, and rational) follow the laws and rules for exponents and the order of operations (BEDMAS).
- Express all answers using positive exponents.
- Express all answers in radical form.

Practise, Apply, Solve 1.10

A

1. Simplify.

(a) $(x^4)(x^3)$ (b) $(c^4)^5$ (c) $x^6 \div x^3$ (d) $(ab)^3$

(e) $(d^4)(d^2)(d^{-3})$ (f) $b^{-3} \div b^2$ (g) $(2x^3)^2$ (h) $\left(\dfrac{2x}{-3y^2}\right)^3$

2. Write each expression as a power of 2.

(a) $2^4 \times 2^5$ (b) $2^n \times 2^m$ (c) $(2^4)^x$ (d) 4^3

(e) 8^n (f) $2^6 \div 2^{-2}$ (g) $4^4 \div 2^m$ (h) 16^{3x-1}

3. Evaluate.

(a) 3^5 (b) 7^0 (c) 8^{-2} (d) $(-3)^4$ (e) $-16^{-\frac{3}{4}}$

(f) $16^{\frac{1}{4}}$ (g) $32^{\frac{2}{5}}$ (h) $4^{\frac{3}{2}}$ (i) $125^{-\frac{1}{3}}$ (j) $100^{-\frac{1}{2}}$

(k) $-27^{\frac{1}{3}}$ (l) $256^{\frac{3}{4}}$ (m) $27^{\frac{2}{3}}$ (n) $128^{-\frac{3}{7}}$ (o) $64^{\frac{2}{3}}$

4. Rewrite using positive exponents.

(a) $(x^{-3})^2$ (b) $\sqrt[3]{x}$ (c) $\sqrt[4]{c^5}$ (d) $\sqrt[3]{a^{-2}}$ (e) $(\sqrt{c^{-3}})^2$

5. Simplify.

(a) $\dfrac{(xy)^4}{xy}$ (b) $\dfrac{3(ab)^4}{(-a^2)^2}$ (c) $(a^3b)^2\left(\dfrac{-a}{b}\right)^3$ (d) $(-d^3)^4\left(\dfrac{c}{d}\right)^6$

(e) $\left(\dfrac{-1}{b}\right)^2(a^3b)^2$ (f) $\dfrac{(a^4b^2)^3}{(a^2b^2)^2}$ (g) $\left(\dfrac{x}{-y}\right)^3(-xy)^4$ (h) $\left(\dfrac{x^3}{y^4}\right)^4\left(\dfrac{y^2}{x^4}\right)^3$

B

6. Simplify.

(a) $49^{\frac{1}{2}} + 16^{\frac{1}{4}}$

(b) $27^{\frac{2}{3}} - 81^{\frac{3}{4}}$

(c) $16^{\frac{3}{4}} + 16^{\frac{3}{4}} - 81^{-\frac{1}{4}}$

(d) $128^{-\frac{5}{7}} - 16^{-0.75}$

(e) $16^{\frac{3}{2}} + 16^{-0.5} + 8 - 27^{\frac{2}{3}}$

(f) $81^{\frac{1}{2}} + \sqrt[3]{8} - 32^{\frac{4}{5}} + 16^{\frac{3}{4}}$

(g) $9^{\frac{1}{2}} - \sqrt[4]{16} + 16^{\frac{1}{2}} - 2(2^{-3})$

(h) $\left(\frac{1}{8}\right)^{\frac{1}{3}} - \sqrt[3]{\frac{27}{125}} + 4\left(8^{-\frac{2}{3}}\right)$

7. Evaluate using the laws of exponents.

(a) $2^3 \times 4^{-2} \div 2^2$

(b) $(2^2 \times 3)^{-1}$

(c) $\left(\frac{3^{-1}}{2^{-1}}\right)^{-2}$

(d) $4^{-1}(4^2 + 4^0)$

(e) $\frac{2^5}{3^{-2}} \times \frac{3^{-1}}{2^4}$

(f) $(5^0 + 5^2)^{-1}$

(g) $\frac{3^{-2} \times 2^{-3}}{3^{-1} \times 2^{-2}}$

(h) $\frac{4^{-2} + 3^{-1}}{3^{-2} + 2^{-3}}$

(i) $\frac{5^{-1} - 2^{-2}}{5^{-1} + 2^{-2}}$

8. Rewrite each expression, without using fractions.

(a) $\frac{x}{y^2}$

(b) $\frac{m^3}{n^2}$

(c) $\frac{2x^3}{y^5z^2}$

(d) $\frac{10a^2b}{2c^2}$

(e) $\frac{30m^2}{-6n^4}$

(f) $\frac{1}{x^{-2}}$

(g) $\frac{m^{-5}}{n^{-4}}$

(h) $\frac{8x^3y^{-2}}{4z^{-3}}$

9. Express using only positive exponents.

(a) a^4b^{-3}

(b) $\frac{4a^{-2}b^3}{c^{-2}}$

(c) $\frac{5a^{-2}b^{-3}}{ab^{-2}}$

(d) $\frac{2a^2b^{-3}}{3bc^{-4}}$

(e) $\frac{(4a)^{-2}}{(3b)^{-1}}$

10. Find the value of each expression for $a = 1$, $b = 3$, and $c = 2$.

(a) ab^c

(b) a^cb^c

(c) $(ab)^{-c}$

(d) $(b \div c)^{-a}$

(e) $(-a \div b)^{-c}$

(f) $(a^{-1}b^{-2})^c$

(g) $(a^bb^a)^c$

(h) $[(b)^{-a}]^{-c}$

11. Simplify and then determine the number or numbers represented by each variable.

(a) $n^3 = 64$

(b) $c^3 = 512$

(c) $d^4 = 625$

(d) $(n^2)^5 \div n^5 = 243$

(e) $x^{15} \div x^2 = 12^{13}$

(f) $(t^2)^2 \times t^0 = 10\ 000$

(g) $m^3 \times m^2 = 3125$

(h) $\left(\frac{m^7}{m^6}\right)^2 = 196$

(i) $n^{10} \div n^8 = 576$

(j) $(x^8)(x^2) \div (x^7) = 1$

(k) $(p \times p^2)^3 = 3^9$

(l) $(x^2)^2 \div x^3 = 15$

12. State whether each expression is true or false.

(a) $9^{\frac{1}{2}} + 4^{\frac{1}{2}} = (9 + 4)^{\frac{1}{2}}$

(b) $\left(9^{\frac{1}{2}}\right)\left(4^{\frac{1}{2}}\right) = (9 \times 4)^{\frac{1}{2}}$

(c) $\left(\frac{1}{a} + \frac{1}{b}\right)^{-1} = a + b$

(d) $\left(\frac{1}{a} \times \frac{1}{b}\right)^{-1} = ab$

(e) $\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^6 = x^2 + y^2$

(f) $\left[\left(x^{\frac{1}{3}}\right)\left(y^{\frac{1}{3}}\right)\right]^6 = x^2y^2$

13. Simplify.

(a) $\sqrt{x^2}$ (b) $\sqrt[3]{c^3}$ (c) $\sqrt[3]{z^9}$

(d) $\sqrt[4]{d^8}$ (e) $\left(\frac{2}{3}\right)^{\frac{5}{3}}\left(\frac{2}{3}\right)^{\frac{1}{3}}$ (f) $\left(4^{\frac{1}{3}}\right)^{\frac{6}{4}}$

(g) $\left(2x^{-\frac{1}{3}}y^{\frac{3}{4}}\right)^2$ (h) $\left(-3u^{\frac{3}{5}}v^{-\frac{1}{5}}\right)^2$ (i) $\dfrac{18y^{\frac{4}{3}}z^{-\frac{1}{3}}}{24y^{-\frac{2}{3}}z}$

(j) $\dfrac{a^{\frac{3}{4}} \times a^{\frac{1}{2}}}{a^{\frac{3}{2}}}$ (k) $\left(c^{\frac{3}{2}}\right)^{\frac{1}{3}}$ (l) $\left(k^{-\frac{1}{2}}\right)^{\frac{2}{5}}$

14. Simplify.

(a) $\dfrac{x^{\frac{1}{2}} \times x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$ (b) $\dfrac{x^{\frac{5}{6}} \times x^{\frac{2}{3}}}{x^{\frac{1}{2}}}$ (c) $\left(y^{\frac{1}{2}}\right)^2 \div (16y^6)^{\frac{1}{2}}$

(d) $\left(\dfrac{\sqrt[4]{y^4}}{\sqrt{y^2}}\right)^3$ (e) $\left(\dfrac{x^3}{81}\right)\left(\dfrac{81^{\frac{3}{4}}}{x}\right)$ (f) $\dfrac{(x^2y^4)^{\frac{1}{2}}(x^4y^2)^{\frac{1}{2}}}{\left(x^{\frac{1}{2}}y^{\frac{1}{2}}\right)^6}$

15. Simplify.

(a) $(x^2)^{5-r}$ (b) $(a^{4+2r})(a^{-3r-5})$

(c) $(b^{2m+3n}) \div (b^{m-n})$ (d) $x^{3(7-r)}x^r$

(e) $(a^{10-p})\left(\frac{1}{a}\right)^p$ (f) $[(3x^4)^{6-m}]\left(\frac{1}{x}\right)^m$

16. Simplify.

(a) $(3x^3)^2$ (b) $(4x^2y^{-4})^{-2}$ (c) $(16x^{10}y^4)^{\frac{1}{2}}$

(d) $(2x^{-2})^2(3x^4)^{-3}$ (e) $\dfrac{(2x^{-1})^{-2}}{2(y^{-1})^{-2}}$ (f) $\left(\dfrac{3x^2}{y^{-1}}\right)^{-2}\left(\dfrac{2y^2}{3x}\right)^3$

17. **Check Your Understanding:** Simplify $\left(\dfrac{3x^3}{2y^4}\right)^2\left(\dfrac{2y^2}{3x^4}\right)^{-3}$.

C

18. Solve.

(a) $x^3 = 27$ (b) $x^{\frac{1}{2}} = 5$ (c) $c^{\frac{2}{3}} = 64$ (d) $3a^{\frac{4}{5}} = 48$

19. Solve.

(a) $\left(\frac{1}{16}\right)^{\frac{1}{4}} - \sqrt[3]{\frac{8}{27}} = \sqrt{x^2}$ (b) $\sqrt[3]{\frac{1}{8}} - \sqrt[4]{x^4} + 15 = \sqrt[4]{16}$

20. If $a = 2$ and $b = -1$, which expression has the greater value?

A: $\dfrac{a^{-2b}a^{-b+2}}{(a^{-2})^b}$ B: $\dfrac{(a^b)^{-3}a^{-(1-2b)}}{(a^{-b})^3}$