Chapter 6 Review

Extending Skills with Trigonometry

Check Your Understanding

1. Explain why the sine law holds true for obtuse angle triangles as well as acute angle triangles.

2. What dimensions of a triangle do you need to know to use the cosine law? the sine law?

3. If you are given a side, an angle, and the side opposite this angle, how many triangles could you create? What additional information about the triangle do you need to help you decide which case you are dealing with?

4. You are asked to determine the exact value of $\sin \frac{\pi}{4}$.

(a) Explain why you cannot use a calculator in this case.

(b) What must you use to accomplish this?

(c) To obtain an approximate value from the calculator, what mode would you use?

5. Determine, over the domain $0 \leq \theta \leq 2\pi$, the maximum number of solutions possible for a trigonometric equation based on

(a) the sine function with a period of 2π

(b) the sine function with a period of 4π

(c) the tangent function with a period of π

6. How many solutions does a trigonometric equation have if the domain is not specified? Explain why.

7. If an equation is also an identity, how many solutions can it have? Explain.

8. Explain how you could prove that a given equation is an identity using

(a) a graph

(b) algebra

9. Write an example of

(a) a linear trigonometric equation

(b) a quadratic trigonometric equation

10. Can all quadratic trigonometric equations be solved by factoring? Explain.

11. Approximate solutions to linear and quadratic trigonometric equations can be found using graphing technology. What must be done to the equation so that the zeros of the corresponding trigonometric function can be used to solve the original equation?

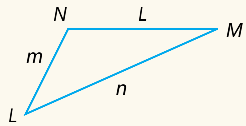
Review and Practice

6.1 Extending Trigonometry Skills with Oblique Triangles

1. Complete this table based on the given information for an oblique triangle.

Given Information	What Can Be Found	Law Required
two angles and any side (AAS or ASA)		
two sides and the contained angle (SAS)		
three sides (SSS)		
two sides and an angle opposite one of them (SSA)		

2. For the triangle shown,



(a) Write the sine law you could use to find $\angle L$.

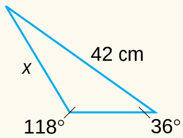
(b) Write the cosine law you could use to find m.

3. In ΔABC, $a = 8$ cm and $\angle A = 62°$. Find a value for b so that $\angle B$ has

(a) two possible values (b) one possible value (c) no possible value

4. Determine the measure of the indicated side or angle. Answer to one decimal place.

(a)



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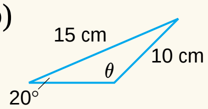

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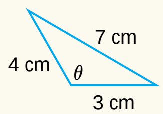

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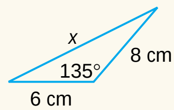


5. Solve each triangle.

(a) In ΔABC, $a = 12$ cm, $c = 11$ cm, and $\angle B = 115°$.

(b) In ΔQRS, $\angle Q = 48°$, $\angle R = 25°$, and $s = 121$ cm.

6. In ΔABC, $a = 13.4$ cm, $b = 16.6$ cm, and $\angle A = 38°$. Determine how many solutions ΔABC has. Then solve ΔABC.

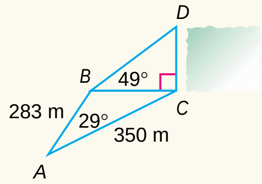
6.2 Solving Trigonometry Problems in Two and Three Dimensions

7. Name three things to consider when solving a problem involving trigonometry. What should be part of every solution to a trigonometry problem?

8. A surveyor places a base line along one bank of a river. From each end of the base line, a rock is sighted on the opposite bank of the river. The base line is 160 m long and the lines of sight of the rock form angles of 42° and 71° with the base line. How wide is the river?

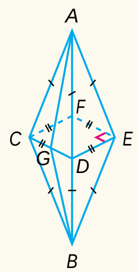
9. The radar operator at an airport control tower locates two planes flying toward the airport at the same altitude. One plane is 120 km from the airport at a bearing of N70°E. The other is 180 km away, on a bearing of S55°E. How far apart are the planes?

10. An engineer needs to find the height of an inaccessible cliff and takes the measurements shown. How high is the cliff?



11. Two roads intersect at 15°. Two cars leave the intersection, each on a different road. One car travels at 80 km/h and the other car at 100 km/h. After 30 min, a traffic helicopter, 750 m directly above and between the cars, notes the angle of depression of the slower car is 18°. What is the horizontal distance from the helicopter to the faster car?

12. In the octahedron shown, $AB = 25$ cm, $AC = 15$ cm, and $CD = 10$ cm. Determine the size of $\angle AGB$.



6.3 Using Special Triangles to Determine Exact Values

13. Draw a well-labelled diagram of a right triangle with an angle of 45°. Use the triangle to determine

(a) $\cos 45°$ (b) $\sin 135°$ (c) $\cos 225°$ (d) $\tan(-45°)$

14. Draw a well-labelled diagram of a right triangle with an angle of $\frac{\pi}{6}$. Use the triangle to determine

(a) $\sin \frac{\pi}{6}$ (b) $\cos \frac{\pi}{6}$ (c) $\tan \frac{5\pi}{6}$ (d) $\sin \frac{5\pi}{3}$

15. Calculate each value exactly.

(a) $\sin^2(45°) + \cos^2(60°)$

(b) $\sin \frac{-\pi}{4} \cos \frac{5\pi}{4} + \tan^2 \frac{\pi}{4}$

(c) $\cos(60°) \sin^2(240°) - \sin(-60°)$

(d) $3\sin^2 \frac{\pi}{3} - 2\cos^2 \frac{\pi}{6}$

16. Prove that $\tan \frac{3\pi}{4} + \frac{1}{\tan \frac{3\pi}{4}} = \frac{1}{\sin \frac{3\pi}{4} \cos \frac{3\pi}{4}}$.

17. For each equation, find possible values of x, $0° \le x \le 360°$.

(a) $\sin x = \frac{\sqrt{3}}{2}$ (b) $-2\cos 2x = 1$ (c) $\sqrt{3}\tan x = 1$

6.4–6.5 Trigonometric Identities

18. Write an example of a trigonometric identity.

19. How many solutions exist for the identity that you wrote in question 18? Explain.

20. Prove that $\tan x \sin x + \cos x = \frac{1}{\cos x}$.

21. Prove that $\tan x = \frac{\sin x + \sin^2 x}{\cos x\,(1 + \sin x)}$.

22. Prove that $\sin^4 x - \cos^4 x = \sin^2 x - \cos^2 x$.

23. Prove that $\frac{1 - \tan^2 \theta}{\tan \theta - \tan^2 \theta} = \frac{\tan \theta + 1}{\tan \theta}$.

6.6 Solving Quadratic Trigonometric Equations

24. How can you identify whether or not a trigonometric equation is quadratic?

25. What algebraic technique can sometimes be used to solve a quadratic trigonometric equation?

26. Solve for x, where, $0° \le x \le 360°$.

(a) $\cos x\,(1 - 2 \sin x) = 0$

(b) $4 \sin^2 x - 1 = 0$

(c) $2 \cos^2 x + \cos x - 1 = 0$

(d) $3 \sin^2 x + 4 \sin x = 4$

(e) $1 - \cos x = 4 \sin^2 x$

(f) $5 \cos^2 x - 3 = 0$

27. Solve for θ, where $0 \le \theta \le 2\pi$.

(a) $(\sin \theta + 1)(\cos \theta - 1) = 0$

(b) $\cos^2 \theta = \frac{1}{4}$

(c) $\cos^2 2\theta = \cos 2\theta$

(d) $10 \sin^2 \theta + 7 \sin \theta = 6$

(e) $\tan^2 \theta = \sqrt{3}$

(f) $\cos^2 \theta - 5 \sin \theta + 6 = 0$

Chapter 6 Summary

In this chapter, you saw that the sine law and the cosine law can be used to solve oblique triangles, and you used these laws to solve trigonometric problems in two and three dimensions. You also saw what an identity is and how to verify an identity. You also used skills for solving quadratic equations and linear trigonometric equations to solve a new type of equation, a quadratic trigonometric equation.

Chapter 6 Review Test

Extending Skills with Trigonometry

1. Solve each triangle. Round each value to the nearest tenth of a unit.

(a) ΔLMN; $\angle L = 75°$, $m = 110$ m, and $n = 95$ m

(b) ΔABC; $\angle A = 108°$, $b = 15.5$ cm, and $a = 20.9$ cm

2. **Knowledge and Understanding**
Solve ΔABC, where $\angle A = 38°$, $b = 25$ cm, and $a = 21$ cm

3. Determine each value exactly.

(a) $\sin \frac{\pi}{6}$ (b) $\cos 135°$

(c) $\tan \frac{-5\pi}{6}$ (d) $\sin 300°$

4. Solve for x, where $0° \leq x \leq 360°$.

(a) $2 \cos x = \sqrt{2}$

(b) $\sin^2 x = \frac{1}{4}$

5. Prove each identity.

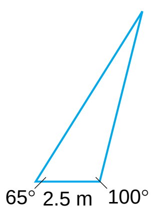
(a) $\tan 2\beta = \frac{(1 - \cos \beta)(1 + \cos \beta)}{(1 - \sin \beta)(1 + \sin \beta)}$

(b) $\sin \theta + \tan \theta = \tan \theta\,(1 + \cos \theta)$

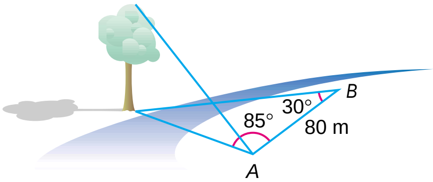
6. Solve $2 \cos^2 x - 13 \cos x = -6$, where $0° \leq x \leq 360°$.

7. Solve $1 - 2 \sin^2 \theta = -\cos \theta$, where $0 \leq \theta \leq 2\pi$.

8. **Application:** Find the area of this triangle.

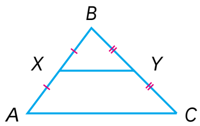


9. Bert wants to find the height of a tree across a river. To do this he lays out a base line 80 m long and measures the angles as shown. The angle of elevation from A to the top of the tree is 8°. How tall is the tree?



10. **Communication:** Explain when one of the factors of a quadratic trigonometric equation will not lead to a solution of the original equation. Use an example in your explanation.

11. **Thinking, Inquiry, Problem-Solving**
In the diagram, points X and Y are the midpoints of AB and BC, respectively.



Show that $AC = 2XY$.