Chapter 5 Review

Modelling Periodic Functions

Check Your Understanding

1. How can you tell when the graph of a set of data is periodic?

2. List five real-life situations that can be modelled by periodic functions. Explain why they are periodic.

3. (a) Sketch a sinusoidal curve.
(b) Name the primary trigonometric functions that are models of sinusoidal curves.
(c) How are the functions the same?
(d) How are the functions different?

4. Consider a set of data that can be modelled using a sinusoidal function. Describe the relationship between
(a) the maximum and minimum values and the axis of the function
(b) the maximum and minimum values and the amplitude of the curve

5. Explain the following sentence: Sinusoidal functions have period lengths that are multiples of 360° or 2π.

6. (a) Define the meaning of radian measure and explain its use.
(b) Describe how to convert between degree measure and radian measure.

7. Point $P(x, y)$ is on the terminal arm of an angle θ in standard position. State the primary trigonometric functions.

8. (a) Explain why the tangent function is not defined for all values of the independent variable.
(b) What occurs at the point where the tangent function is not defined? How does this feature help when sketching the graph?

9. Describe the transformation on $y = \cos\theta$ for each function. Use a sketch in your description.
(a) $y = \cos 2\theta$ (b) $y = -\frac{1}{2}\cos\theta$ (c) $y = \cos\frac{\theta}{2}$
(d) $y = \cos\theta + 2$ (e) $y = \cos(\theta + 2)$

10. What transformations are needed to obtain the graph of $y = a\sin k(\theta + b) + d$ from the graph of $y = \sin\theta$ under each set of conditions?
(a) $a > 1, k > 1, b < 0, d > 0$ (b) $-1 < a < 0, 0 < k < 1, b > 0, d < 0$

11. Explain how to use the domain of a periodic function to determine all values of θ when $\sin\theta = 0.5$.

Review and Practice

5.1 Periodic Phenomena

1. What feature of a function determines whether or not it is periodic?

2. Explain what each item means for a periodic curve. Show each item on a labelled diagram.

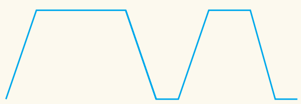
(a) cycle
(b) period
(c) amplitude
(d) axis of the curve
(e) maximum and minimum

3. Explain why each case could be an example of a periodic phenomenon.

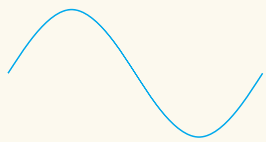
(a) the number of litres of ice cream consumed in Ontario each year
(b) the number of cubic metres of natural gas used each year by a factory
(c) the position of a pendulum in a grandfather clock
(d) the vibration of a guitar string
(e) the average monthly low temperature in London
(f) the sound of a fog horn

4. Sketch two cycles for each periodic relation.

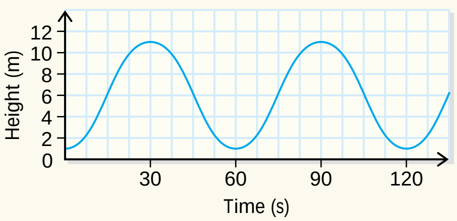
(a) wash/rinse cycle of a dishwasher: period 10 min, amplitude 10 L, and axis 10 L



(b) tip of a windmill blade above the ground: period 50 s, amplitude 2 m, and axis 3 m



5. The graph shows Nina's height above the ground as she rides the Ferris wheel at the fair.



(a) State the maximum and minimum heights of the ride.
(b) How long does the wheel take to make one revolution?
(c) What is the amplitude of the curve? How does this relate to the Ferris wheel?
(d) Write an equation to represent the axis of the curve.

5.2 Understanding Angles

6. Explain each term.

(a) angle in standard position (b) positive angle (c) negative angle
(d) coterminal angle (e) principal angle (f) related acute angle

7. (a) How many degrees are possible for a positive angle, θ, that terminates in quadrant I? quadrant II? quadrant III? quadrant IV?

(b) How many radians are possible for a positive angle, θ, that terminates in quadrant I? quadrant II? quadrant III? quadrant IV?

8. Draw each angle. Show the principal angle, its measure, and the measure of the related acute angle.

(a) $-130°$ (b) $500°$ (c) $-560°$ (d) $280°$

9. State all values of θ in each case.

(a) $\theta = 62° + 360°n$, for $-2 \leq n \leq 3$, $n \in \mathrm{I}$

(b) $\theta = -137° + 360°n$, for $-1 \leq n \leq 2$, $n \in \mathrm{I}$

10. State the measure of all coterminal angles when

(a) the principal angle is 110° and $-360° \leq \theta \leq 540°$

(b) the principal angle is 190° and $-540° \leq \theta \leq 720°$

11. Point $P(-11, 14)$ is on the terminal arm of an angle θ in standard position.

(a) Mark the principal angle on a sketch. In which quadrant does the angle terminate?

(b) Determine the measure of the related acute angle to the nearest degree.

(c) What is the measure of θ to the nearest degree?

5.3 Trigonometric Functions

12. (a) State the primary trigonometric functions for any point $P(x, y)$ on the terminal arm of an angle θ in standard position.

(b) Explain how point $P(x, y)$ can be written with coordinates $P(r\cos\theta, r\sin\theta)$.

13. (a) Sketch the graph of $y = \sin\theta$ for $-360° \leq \theta \leq 360°$.

(b) State the period, amplitude, range, and axis of the curve.

(c) State the coordinates of all maximum values, minimum values, and zeros of the curve within this domain.

14. Repeat question 13 for $y = \cos\theta$.

15. (a) Sketch the graph of $y = \tan \theta$ for $-360° \leq \theta \leq 360°$.

(b) Why is the function undefined for certain values of θ?

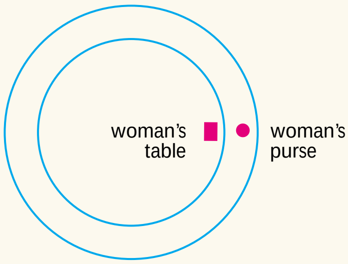
(c) What occurs at the point where the graph is undefined?

16. Point $(-17, -20)$ is on the terminal arm of an angle θ in standard position.

(a) State the primary trigonometric functions for θ.

(b) Determine the measure of θ to the nearest degree.

17. A circular dining room at the top of a skyscraper rotates in a counterclockwise direction so diners can see the entire city. A woman sits next to the window ledge and places her purse on the ledge as shown. Eighteen minutes later she realizes that her table has moved but her purse is on the ledge where she left it. The coordinates of her position are $(x, y) = (20 \cos (7.5t)°, 20 \sin (7.5t)°)$, where t is the time in minutes and x and y are in metres. What is the shortest distance she has to walk to retrieve her purse? Round to one tenth of a metre.



5.4–5.5 Radian Measure

18. How are degree measure and radian measure alike? different?

19. Show that $\theta = 2\pi$ for one revolution of a circle with radius r.

20. Convert each degree to exact radian measure and then evaluate to one decimal.

(a) 20° (b) −50° (c) 160° (d) 420° (e) −280°

21. Convert each radian measure to degree measure.

(a) $\frac{\pi}{4}$ (b) $-\frac{5\pi}{4}$ (c) $\frac{8\pi}{3}$ (d) $-\frac{2\pi}{3}$ (e) $\frac{11\pi}{6}$

22. Round each radian measure to the nearest degree and mark it on the unit circle.

(a) 3.2 (b) −1.4 (c) 8.3 (d) 1.5 (e) 2.2

23. Graph $y = \sin \theta$, for $-\frac{3\pi}{2} \leq \theta \leq \frac{5\pi}{2}$.

24. Determine θ to one decimal place for $-2\pi \leq \theta \leq 2\pi$.

(a) $\sin \theta = 0.75$ (b) $\cos \theta = -0.35$ (c) $\tan \theta = 6.33$

25. A ship that is docked in port and rises and falls with the waves. The model $h(t) = \sin\left(\frac{\pi}{5}\right)t$ describes the vertical movement of the ship, h, in metres at t seconds.

(a) What is the vertical position of the ship at 22 s to the nearest hundredth of a metre?

(b) What is the period of the function? What does this mean in this case?

(c) Determine all times within the first minute that the vertical position of the ship is -0.9 m to the nearest tenth of a second.

5.6 Investigating Transformations

26. Explain how to use transformations to sketch the graph of $y = a \sin k(\theta + b) + d$.

27. The average monthly temperature, T, of a town is given by $T(t) = -24 \cos \frac{\pi}{6}t + 9$.

(a) Determine the period and explain what it means in this case.

(b) How can you use the maximum and minimum values of the function $y = \cos \theta$ to determine the maximum and minimum values of T?

28. Points $\left(\frac{\pi}{2}, 1\right)$ and $(\pi, 0)$ are on the curve $y = \sin \theta$. State the new coordinates under each transformation.

(a) $y = -3 \sin \theta$

(b) $y = \sin \theta + 1$

(c) $y = \sin\left(\theta + \frac{\pi}{4}\right)$

(d) $y = \sin 2\theta$

29. Sketch the graphs using transformations of the base curve for $0 \le \theta \le 2\pi$.

(a) $y = 2 \sin 3\left(\theta - \frac{\pi}{6}\right) - 4$

(b) $y = -3 \cos\left(2\theta + \frac{\pi}{2}\right) + 2$

30. State the amplitude, period, phase shift and vertical shift.

(a) $y = 2 \cos\left(\frac{\theta}{2} - 60°\right) + 1$

(b) $y = -3 \sin (2\theta + 90°) - 1$

(c) $y = 1 + 2 \cos 3\left(\theta - \frac{\pi}{6}\right)$

5.7 Modelling Periodic Phenomena

31. (a) Why should a set of data be graphed before trying to fit it to a trigonometric model?

(b) List the four pieces of information needed to complete the general form of a trigonometric equation. Explain how to determine this information.

32. Determine a trigonometric model to represent each set of data.

(a)

θ	0°	30°	60°	90°	120°
$f(\theta)$	0	4	0	−4	0

(b)

t	−2	0	2	4	5.7	8.0	10.0	12
$f(t)$	5.5	6.0	5.5	4.5	4.0	4.5	5.5	6

33. The Double Scoop Ice Cream Company tracked its mean monthly production of ice cream over the last two years.

Ice Cream Production in Thousands of Litres

Month	J	F	M	A	M	J	J	A	S	O	N	D
Year 1	168	181	219	222	246	276	264	252	219	204	181	174
Year 2	169	180	220	221	245	274	265	251	219	203	180	175

(a) Explain why it is reasonable to expect ice cream production to be periodic.

(b) Determine a trigonometric model that best represents the data.

(c) Use a graphing calculator to graph your trigonometric model. Comment on the closeness of fit to the given data.

5.8 Solving Linear Trigonometric Equations

34. Identify three different ways that an approximate solution to a linear trigonometric equation can be determined.

35. Determine whether the given value is a solution to the equation.

(a) $\cos\theta = \frac{1}{2}, \theta = \frac{5\pi}{3}$ (b) $5\sin\theta + 6 = 10, \theta = 25°$

36. Solve for x to one tenth of a degree, where $0° \leq x \leq 360°$.

(a) $\sin x = \frac{2}{3}$ (b) $\cos x = -\frac{5}{8}$ (c) $\tan x = 2.8$

(d) $3\sin 2x = -4$ (e) $5\cos(0.5x) - 1 = 1$ (f) $3\tan\frac{x}{2} + 2 = 9$

37. Solve for x to one hundredth of a radian, where $0 \le x \le 2\pi$.

(a) $\cos x = \frac{7}{8}$ (b) $\sin x = -\frac{3}{11}$ (c) $\tan 2x = 1$

(d) $5 \sin x = -5$ (e) $5 \cos 3x - 1 = 1$ (f) $3 \tan \frac{x}{2} + 2 = 9$

38. Solve for x to one tenth of a degree, where $0° \le x \le 360°$. Use graphing technology.

(a) $10 \sin x - 3 = 5$ (b) $4 \cos x = -1$

(c) $2 \tan x - 1 = 5$ (d) $2 \sin (2x + 90°) = 1$

39. Solve for x to one hundredth of a radian, where $0 \le x \le 2\pi$. Use graphing technology.

(a) $\tan x + 7.5 = 8$ (b) $3 \cos (3x) = -3$

(c) $4 \sin (3x + \pi) + 1 = -3$ (d) $-\cos (2x + 2\pi) - 1 = 0$

40. The average monthly maximum temperature of Windsor can be modelled by $T(t) = 14.9 \sin \frac{\pi}{6}(t - 3) + 13$, where T is the temperature in Celsius and $t = 0$ represents January 1, $t = 1$ represents February 1, and so on.

(a) When is the average monthly maximum temperature highest? lowest?

(b) Use the model to predict when the temperature is 0°C.

(c) When does the temperature reach 25°C?

Chapter 5 Summary

In this chapter, you saw that a periodic function can be represented by a curve with a repeating pattern or cycle. Periodic phenomena occur in the world around us. The life cycle of plants and animals, phases of the moon, motion of waves, and vibration of sound are all examples of periodic phenomena.

Motion about a circle produces trigonometric functions that relate a point's position on the circle to its angle of rotation. These functions are the sine, cosine, and tangent functions, and are called the primary trigonometric functions. They have many real-life applications. The angles of rotation can be expressed in degree measure or radian measure. Radian measure is used when the position is a function of time or distance rather than degrees of rotation.

Trigonometric functions are transformed similarly to how quadratic functions are transformed. The equation $y = a \sin k(\theta + b) + d$ is a transformation of the base curve $y = \sin \theta$. The equation $y = \cos k(\theta + b) + d$ is a transformation of the base curve $y = \cos \theta$.

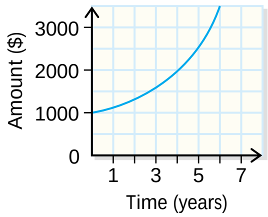
A graphical model of a situation can be drawn if enough data points are known. Once the data is seen to be periodic, an equation can be found for it. A graphing calculator can be used to plot the data points and graph the equation. As well, the SinReg feature of the calculator can be used to determine the equation of periodic data.

Chapter 5 Review Test

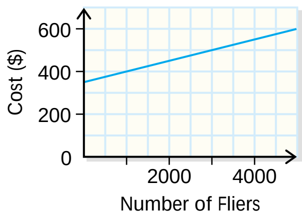
Modelling Periodic Functions

1. Identify each function as linear, quadratic, exponential, or periodic. State the period and the maximum and minimum values of each periodic function.

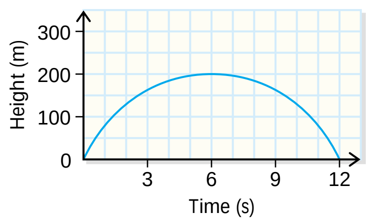
(a) \$1000 is invested at 10% per annum compounded semiannually for 5 years.



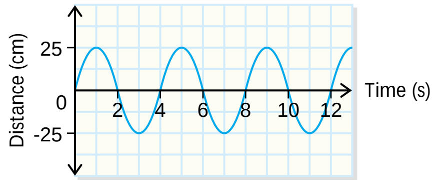
(b) The cost of printing an advertising flier is \$350, plus \$0.05 per copy.



(c) A model rocket is shot vertically upward and falls back to the ground.



(d) A rider on a merry-go-round moves up and down with the horse as the merry-go-round travels in a circle.



2. **Communication:** You are given a set of data that looks like a sinusoidal function. Explain how you can determine an algebraic model from the graph.

3. State all coterminal angles for θ.

(a) $\theta = 47°$, for $-360° \leq \theta \leq 720°$

(b) $\theta = \frac{2\pi}{3}$, for $-2\pi \leq \theta \leq 2\pi$

(c) $\theta = 15°$, for $0° \leq \theta \leq 1080°$

4. Convert to radian measure as an exact value in simplest terms.

(a) $260°$ (b) $-15°$

5. Convert to degree measure.

(a) $\frac{5\pi}{2}$ (b) $-\frac{7\pi}{2}$

6. Express $27°$ in radian measure to two decimal places.

7. Point $P(-3, 7)$ is on the terminal arm of angle θ in standard position.

(a) State the exact values of the primary trigonometric functions.

(b) Determine the value of θ to the nearest degree.

8. **Knowledge and Understanding**
Consider $y = 2 \sin 3(\theta - 30°) + 1$, for $0° \le \theta \le 360°$.

(a) State the values of the period, phase shift, amplitude, and vertical shift.

(b) Sketch five graphs on separate axes to show the progression from the base curve to the given function.

9. (a) Solve $5 \cos \theta = -0.42$ for θ to the nearest degree, where $0° \le \theta \le 360°$.

(b) Solve $\sin 3\theta = -0.6$ for θ to the nearest degree, where $0° \le \theta \le 240°$.

(c) Solve $2 \tan \left(\theta - \frac{\pi}{2}\right) = 57.3$ for θ to one decimal, where $0 \le \theta \le \pi$.

(d) Solve $4 - 3 \sin \left(\theta - \frac{\pi}{2}\right) = 6.2$ for θ to one decimal, where $0 \le \theta \le 2\pi$.

10. The average daily maximum temperature in Kenora is shown for each month.

Time (months)	J	F	M	A
Temperature (°C)	−13.1	−9.0	−1.1	8.5

Time (months)	M	J	J	A
Temperature (°C)	16.8	21.6	24.7	22.9

Time (months)	S	O	N	D
Temperature (°C)	16.3	9.3	−1.2	−10.2

Source: Environment Canada.

(a) Prepare a scatter plot of the data. Let January represent month 0.

(b) Draw a curve of best fit. Explain why this type of data could be expressed as a periodic function.

(c) State the maximum and minimum values.

(d) What is the period of the curve? Explain why this is appropriate within the context of the question.

(e) Write an equation for the axis of the curve.

(f) What is the phase shift if the cosine function acts as the base curve?

(g) Use the cosine function to write an equation that models the data.

(h) Use the equation to predict the temperature for the 38th month. How could this prediction be confirmed using the table?

11. **Application:** The function $d(\theta) = \frac{v^2}{9.8} \sin 2\theta + 0.5$, models the horizontal distance d, in metres, that a baseball is hit. The distance depends on the initial velocity of the ball, v, in metres per second and the angle, θ, that the ball leaves the bat.

(a) A home run leaves the bat at an angle of 45° and travels 138 m. What is the initial speed of the ball?

(b) What is the angle at which the ball is struck if the ball travels 64 m and leaves the bat at a speed of 28 m/s?

12. **Thinking, Inquiry, Problem Solving**
How many solutions exist for $\sin k\theta = \frac{1}{2}$, where $\{k \in I \mid k \ge 1\}$ and $0 \le \theta \le 2\pi$?