

# Mr. Math's Magical Mathematical Mystery Tours (5M)

By: XXXXXXXX

The formula that 5M uses to determine the profit for their tour is,  $P = -0.6n^2 + 36n - 405$ . In this equation P is profit, n is the number of students in the tour.

The least and the greatest number of students that should be accepted in order to make a profit is, least: 16 and greatest: 44 students. There are two ways to get the two numbers, the first is by using a table and the second is by using a generated graph.

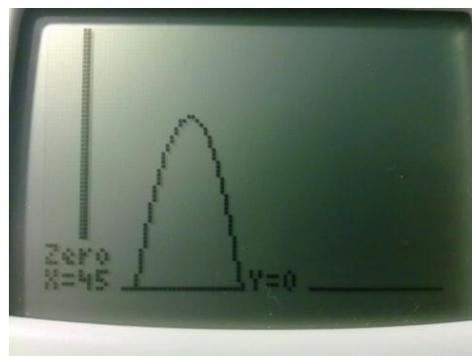
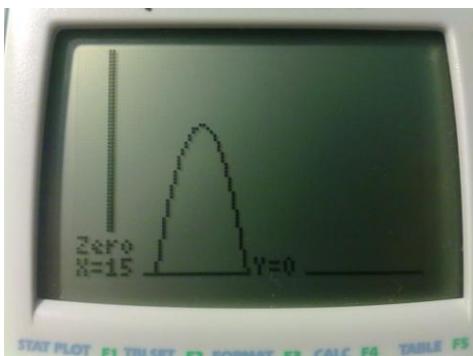
X	Y1
13	-38.4
14	-18.6
15	0
16	17.4
17	33.6
18	48.6
19	62.4

X=16

X	Y1
41	62.4
42	48.6
43	33.6
44	17.4
45	0
46	-18.6
47	-38.4

X=44

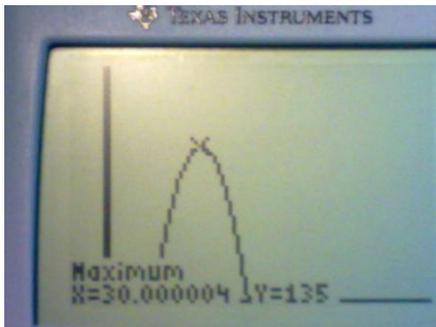
The two photos show the table of the profit formula. X represents the number of students in the tour, and Y represents the profit. As it is shown on the left table, if 15 students go on the tour, the tour breaks even. Starting from the 16<sup>th</sup> student, the tour makes profit. After the 16<sup>th</sup> student the profit increases. On the right photo, starting from the 45<sup>th</sup> students the tour breaks even, so the profit ends at the 44<sup>th</sup> student. After the 45<sup>th</sup> student, the profit starts to decline, and the tour does not make any profit.



The second option that can be used to find the two numbers is by calculating the zeroes on the TI calculator graph. The left graph shows the first zero of the graph (15) and the right graph shows the second zero (45) of the graph. Since on the first and second zero y (profit) equals zero, it is logical to say that the 16<sup>th</sup> and the 44<sup>th</sup> person

will be the least and the greatest number of students that should be accepted in order to make a profit.

The number of students that will give the maximum profit can be determined by getting the vertex points or the axis of symmetry. However, amongst the two ways, the simpler way is using  $\frac{-b}{2a}$  and getting the axis of symmetry. In the equation,  $P = -0.6n^2 + 36n - 405$ ,  $a = -0.6$ ,  $b = 36$ , and  $c = -405$ . This means that,  $\frac{-36}{2(-0.6)} = 30$ . Axis of symmetry equals 30. (In other words 30 equals n) So the number of students that should be accepted to get the maximum profit is 30. The second, but more time consuming way is making a graph and getting the vertex points. (or changing the standard form formula into vertex form, however in this report only the graph way will be performed.)



This graph is based on the profit equation. As it is shown on the graph, the vertex point is (30.0, 135). This means to get the maximum profit (which is 135) 30 students are needed.

It is very simple to get the maximum profit per student. Since the maximum profit of the tour and the number of students to get the maximum profit is determined already in the paragraph above. (Maximum profit: 135, Number of students: 30) The maximum profit per student can be determined by dividing the two values.  $\frac{135}{30} = 4.5$ . Therefore \$4.5 is the maximum profit per person.

The least and the greatest number of students that should be accepted to get the profit of \$96.60 can be determined by inputting 96.60 in to P. This makes the equation,  $96.60 = -0.6n^2 + 36n - 405$ , this equation can be formed as  $-0.6n^2 + 36n - 501.6 = 0$ , n can be determined by inputting this equation into the quadratic equation which is  $\frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$ . The inputted form is,  $\frac{-36 \pm \sqrt{(36)^2 - 4(-0.6)(-501.6)}}{2(-0.6)}$ , if this formula is calculated the two variables for n is 22 and 38. If the two numbers are inputted into the Profit equation, the Profit equals 96.60.

Below is a table showing the relationship between the number of students on the tour and the total amount of profit.

Number of students	0	5	10	15	20	25	30	35	40	45	50
Profit	-405	-240	-105	0	75	120	135	120	75	0	-105

Before 15<sup>th</sup> student the profit is negative, and the tour breaks even at the 15<sup>th</sup> student and the 45<sup>th</sup> student. On the 30<sup>th</sup> student, the price reaches the maximum. After the 45<sup>th</sup> student, the price declines and starts to go down.

The hand drawn graph starts from 15 students for convenience reasons, and it doesn't include any negative points because, the negative point is not necessary to show

the parabola's shape and main points. (The hand drawn graph will be included separately from this report)

By factoring the profit equation the two zeroes can be determined.

$$P = -0.6n^2 + 36n - 405 \rightarrow \text{This is the given equation}$$

$$P = -0.6(n^2 - 60n + 675) \rightarrow -0.6 \text{ is factored out from the original equation}$$

$$P = -0.6(n - 15)(n - 45) \rightarrow \text{The equation is factored completely}$$

$N = 15 \text{ or } 45 \rightarrow$  These are the two zeroes

To confirm the zeroes, quadratic equation can be used. Quadratic equation is  $\frac{-b \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$ . The inputted form of this equation is  $\frac{-36 \pm \sqrt{(36)^2 - 4(-0.6)(-405)}}{2(-0.6)}$ . Because of the  $\pm$  there are two answers to this equation.

$$\frac{-36 \pm \sqrt{(36)^2 - 4(-0.6)(-405)}}{2(-0.6)} \rightarrow \text{The inputted version}$$

$$(36)^2 - 4(-0.6)(-405) = 324 \rightarrow \text{Inside the root is separately calculated}$$

$$-36 + \sqrt{324} = -18 \rightarrow \text{upper part of the fraction calculated}$$

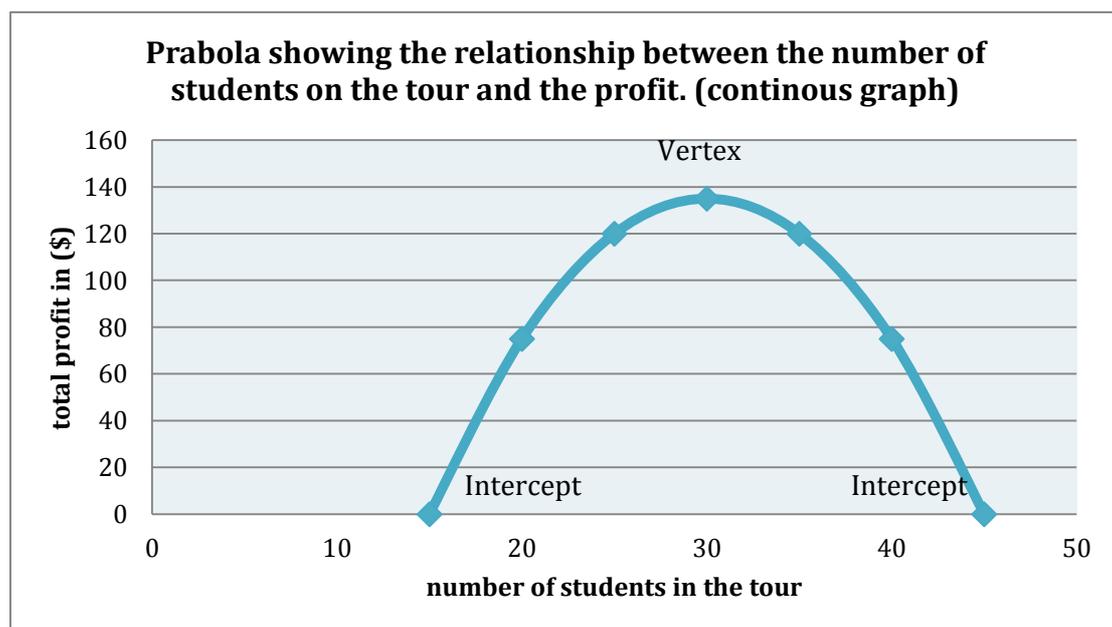
$$\frac{-18}{-1.2} = 15 \rightarrow \text{First zero}$$

$$-36 - \sqrt{324} = -54 \rightarrow \text{Upper part of the fraction for the second equation}$$

$$\frac{-54}{-1.2} = 45 \rightarrow \text{Second zero}$$

As it is shown in the work above, the two zeroes from the factoring, and the two zeroes from the quadratic equation is equal, which means that the two zeroes are correct.

Here is an excel generated graph of the formula  $P = -0.6n^2 + 36n - 405$



Based on all the findings, it is most logical to say that to get the most profit, the number of students that should be accepted in the tour should be 30 students. However the limits can be minimum 16 and maximum 44 students in one tour. With 30 students, the tour benefits the most, however with the number of students within the limit the tour earns profit. With 15 students or 45 students the tour breaks even and earns 0 profit. So getting 15 or 45 students on the tour is meaningless. If the number of students goes below 15 or above 45, there is no profit for the tour at all.