Lesson 8 – Quadratic Modeling

IB Math SL1 - Santowski

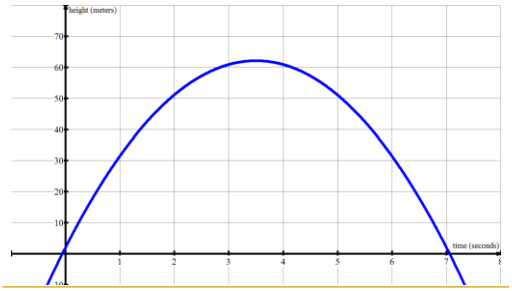
Example 1

- The formula for the height, *h* in meters, of an object launched into the air as a function of its time in flight, *t* in seconds, is given by is $h(t) = -\frac{1}{2} gt^2 + v_o t + h_o$
- g represents the acceleration due to gravity which is about 9.8 m/s^2, v_o refers to the launch velocity in m/s and h_o represents the initial launch height in m.

Example 1

- If a projectile has an initial velocity of 34.3 m/s and is launched 2.1 m above the ground, graphically determine:
- (1) the equation that you will enter into the TI-84
- (2) the time at which the projectile reaches the maximum height
- (3) the maximum height reached by the projectile
- (4) h(2)
- (5) $h^{-1}(12)$
- (6) state the domain and range of the relation and explain WHY
- (7) the x-intercepts and their significance
- (8) the total time of flight of the projectile

Example 1



Example 2

- Determine the flight time of a projectile whose height, *h*(t) in meters, varies with time, *t* in seconds, as per the following formula:
- $h(t) = -5t^2 + 15t + 50$
- (a) Determine a reasonable domain for the function. What does it mean in context?
- (b) What is the range? What does it mean in context?
- (c) Does the projectile attain a height of 70m?
- (d) Determine the maximum height of the projectile?
- (e) When does the object reach this height?
- (f) When does the projectile attain a height of 60 meters?

Example 3

- The path of a baseball thrown at a batter by Mr S is modeled by the equation $h(d) = -0.004d^2 + 0.06d + 2$, where *h* is the height in m and *d* is the horizontal distance of the ball in meters from the batter.
  - (a) what is the maximum height reached by the baseball?
  - (b) What is the horizontal distance of the ball from the batter when the ball reaches its maximum height?
  - (c) How far from the ground is the ball when I release the pitch?
  - (d) How high above the ground is the ball when the ball reaches the batter if she stands 20 m from the pitcher

Example 4

- The cost per hour of running a bus between Burlington and Toronto is modelled by the function $C(x) = 0.0029x^2 - 0.48x + 142$, where x is the speed of the bus in kilometres per hour, and the cost, C, is in dollars. Determine the most cost-efficient speed for the bus and the cost per hour at this speed.

Example 5

- Sasha wants to build a walkway of uniform width around a rectangular flower bed that measures 20m x 30m. Her budget is $6000 and it will cost her $10/m² to construct the path. How wide will the walkway be?

Example 6

- Student council plans to hold a talent show to raise money for charity. Last year, they sold tickets for $11 each and 400 people attended. Student council decides to raise ticket prices for this year's talent show. The council has determined that for every $1 increase in price, the attendance would decrease by 20 people. What ticket price will maximize the revenue from the talent show?

Example 7

- If you plant 100 pear trees in an acre, then the annual revenue is $90 per tree. If more trees are planted, they generate fewer pears per tree and the annual revenue per tree is decreased by $0.70 for each additional tree planted. Additionally, it costs $7.40 per tree per year for maintaining each tree. How many pear trees should be planted to maximize profit?
- (i) What is the equation for revenue?
- (ii) What is the equation for profit?
- (iii) find the max value for the profit equation

Example 8

- Profit type Q
- The demand function (or price function) for a new product is $p(x) = -5x+39$, where p represents the selling price of the product and x is the number sold in thousands. The cost function is $C(x) = 4x + 30$.
- (a) How many items must be sold for the company to break even?
- (b) What quantity of items sold will produce the maximum profit?

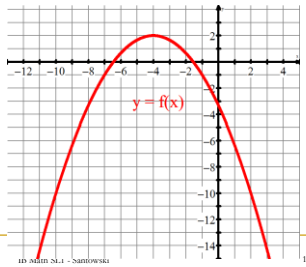
Example 9

For each pair of revenue and cost functions, determine
i. the profit function ii. the value of x that maximizes profit

(a) $R(x) = -x^2 + 24x$, $C(x) = 12x + 28$
(b) $R(x) = -2x^2 + 32x$, $C(x) = 14x + 45$
(c) $R(x) = -3x^2 + 26x$, $C(x) = 8x + 18$
(d) $R(x) = -2x^2 + 25x$, $C(x) = 3x + 17$

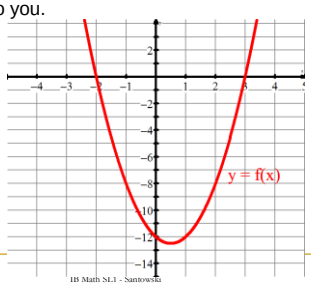
Example 10

- Here are some graphs of quadratic functions ➔ determine their equations in the form that seems most appropriate to you.



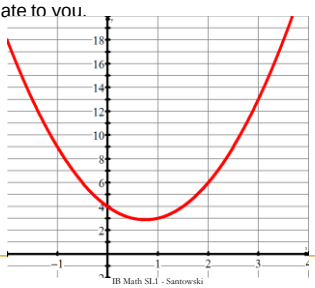
Example 11

- Here are some graphs of quadratic functions ➔ determine their equations in the form that seems most appropriate to you.



Example 12

- Here are some graphs of quadratic functions ➔ determine their equations in the form that seems most appropriate to you.



Example 13

- (1) Write the equation of the parabola that has zeroes of –3 and 2 and passes through the point (4,5).
- (2) Write the equation of the parabola that has a vertex at (4, −3) and passes through (2, −15).
- (3) Write the equation of the parabola that has a y – intercept of –2 and passes through the points (1, 0) and (−2,12).

Example 14

- (1) Write the equation $f(x) = 2(x + 3)^2 - 8$ in standard form
- (2) Write the equation $f(x) = -\frac{1}{2}(x - 5)^2 + 8$ in standard form
- (3) Write the equation $f(x) = 2(x + 3)^2 - 8$ in factored form
- (4) Write the equation $f(x) = -\frac{1}{2}(x - 5)^2 + 8$ in factored form

Example 15

- (1) Write the equation $f(x) = -\frac{1}{2}(x + 4)(x - 2)$ in standard form
- (2) Write the equation $3(x - \frac{1}{2})(x + 3.5)$ in standard form
- (3) Write the equation $f(x) = -\frac{1}{2}(x + 4)(x - 2)$ in vertex form
- (4) Write the equation $3(x - \frac{1}{2})(x + 3.5)$ in vertex form

Example 16

- (1) If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning
- (2) If $y = -4x^2 + kx - 1$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning

Example 17

- Find the range of the parabola $y = -2(x - 4)(x + R)$
- Find the minimum point of $y = x^2 - bx + 4$

Example 18

- Given $f(x) = ax^2 + bx + c$, use the C/S method to rewrite the equation in vertex form, $f(x) = a(x - h)^2 + k$, and thereby determine h and k in terms of a,b & c
- Use the C/S method to rewrite $f(x) = ax^2 + bx + c$ in factored form, $f(x) = a(x - R_1)(x - R_2)$, and thereby determine R_1 and R_2 in terms of a,b, & c.

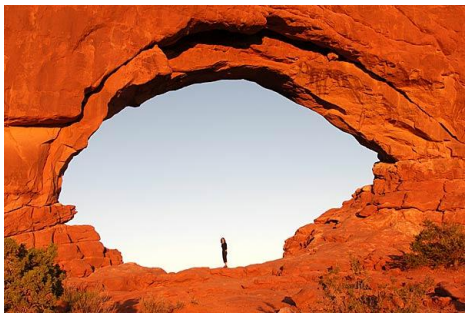
Example 19

- Determine the value of *W* such that $f(x) = Wx^2 + 2x - 5$ has one real root. Verify your solution (i) graphically and (ii) using an alternative algebraic method.
- Determine the value of *b* such that $f(x) = 2x^2 + bx - 8$ has no solutions. Explain the significance of your results.
- Determine the value of *b* such that $f(x) = 2x^2 + bx + 8$ has no solutions.
- Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real roots.
- Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real rational roots.

Example 20

- On the next slide, you will find a picture of my friend who visited Arches National Park a couple of years ago. She is 5 foot 6 inches tall. Determine an equation you can use to model the arch under which she is standing

Photo



Example 21

- On the next slide, you will find a picture of Sydney Harbour Bridge. The bottom "arch" is shaped as a parabola. You will need to research some data about the bridge so that you can determine an equation you can use to model the arch under which she is standing

Photo

Example 22

- Solve the system for *m* such that there exists only one unique solution

$$\begin{cases} y = x^2 + 4x + 6 \\ y = mx + 5 \end{cases}$$

- The line(s) y = mx + 5 are called tangent lines ➔ WHY?
- Now, determine the average rate of change on the parabola (slope of the line segment) between $x_1 = a$ and $x_2 = a + 0.001$ where (a,b) represents the intersection point of the line and the parabola
- Compare this value to *m*. What do you notice?

Example 22