Lesson 7 – Algebra of Quadratics – The Quadratic Formula

IB Math SL1 - Santowski

Fast Five

- Determine HOW many roots the following quadratic functions have:
- (a) $f(x) = x^2 - 6x + 9$
- (b) $f(x) = x^2 - 6x + 5$
- (c) $f(x) = x^2 - 2x + 10$
- (d) $f(x) = -\frac{1}{2}x^2 + 2x + 4$
- (e) $f(x) = x^2 + 2x + c$

Lesson Objectives

- Express a quadratic function in standard form and use the quadratic formula to find its zeros
- Determine the number of real solutions for a quadratic equation by using the discriminant
- Find and classify all roots of a quadratic equation

BIG PICTURE

- Sometimes the same function type can be written in a **variety of different forms** ➔ Sometimes the form of the equation can give us other important information
- **WHY?**
- Is there a **connection** between the form that the **equation** is written in and some of the key features of the **graphs**????

(A) Skills Review

- Given the following quadratic functions, determine the zeroes of the function using the QF (exact and rounded to 3 sf):
- (a) $f(x) = x^2 - 6x + 6$
- (b) $f(x) = 3x^2 + 12x + 9$
- (c) $g(x) = -\frac{1}{4}x^2 + 2x + 3$
- (d) EXPLAIN what you would predict the graphs of these quadratic functions to look like

(A) Skills Review

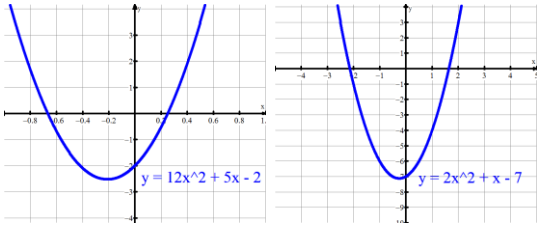
- The quadratic formula looks like:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(B) HOW Does the QF Work ➔ Examples

- Solve $12x^2 + 5x - 2 = 0$ using the Q/F. Then rewrite the equation in factored form and in vertex form. PREDICT the appearance of the graph and its key features
- Determine the roots of $f(x) = 2x^2 + x - 7$ using the Q/F. Then rewrite the equation in factored form and in vertex form. PREDICT the appearance of the graph and its key features

(B) HOW Does the QF Work ➔ Examples



(B) HOW Does the QF Work ➔ Examples

- Solve the system

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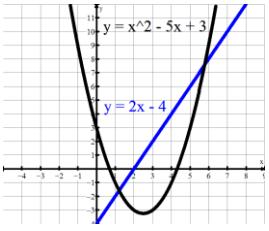


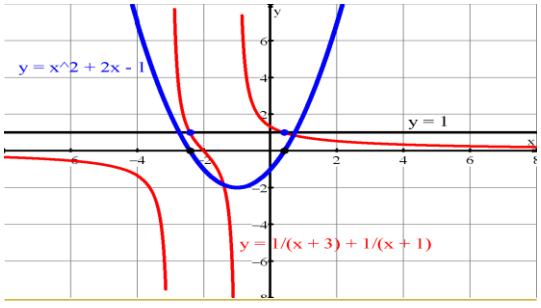
(B) HOW Does the QF Work ➔ Examples

- Solve the equation and graphically verify the 2 solutions

$$\frac{1}{x+3} + \frac{1}{x+1} = 1$$

- Find the roots of $9(x - 3)^2 - 16(x + 1)^2 = 0$
- Solve $6(x - 1)^2 - 5(x - 1)(x + 2) - 6(x + 2)^2 = 0$

(B) HOW Does the QF Work ➔ Examples



(C) WHY Does QF Work ➔ Solving Equations using C/S

- Solve $0 = ax^2 + bx + c$ by completing the square

(C) WHY Does QF Work ➔ Solving Equations using C/S

- If you solve $0 = ax^2 + bx + c$ by completing the square, your solution should look familiar:
- $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- Which we know as the quadratic formula

(C) WHY Does QF Work ➔ Solving Equations using C/S

- Here are some other rearrangements of the QF:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 - \frac{c}{a}}$$

(C) WHY Does QF Work ➔ Examples

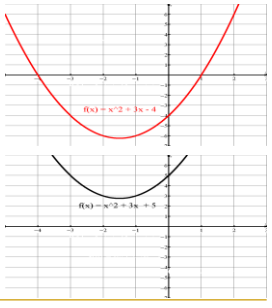
- Given the quadratic function $f(x) = x^2 - 10x - 3$, determine the distance between the roots and the axis of symmetry.
- Determine the distance between the roots and the axis of symmetry of $f(x) = 2x^2 - 5x + 1$

(D)WHY Do We Use Q/F ➔ The Discriminant

- Within the Q/F, the expression $b^2 - 4ac$ is referred to as the discriminant
- We can use the discriminant to classify the "nature of the roots" ➔ a quadratic function will have either 2 distinct, real roots, one real root, or no real roots ➔ this can be determined by finding the value of the discriminant
- The discriminant will have one of 3 values:
  - $b^2 - 4ac > 0$ ➔ which means ➔
  - $b^2 - 4ac = 0$ ➔ which means ➔
  - $b^2 - 4ac < 0$ ➔ which means ➔

(D)WHY Do We Use Q/F ➔ The Discriminant

- Determine the value of the discriminants in:
  - (a) $f(x) = x^2 + 3x - 4$
  - (b) $f(x) = x^2 + 3x + 2.25$
  - (c) $f(x) = x^2 + 3x + 5$



(E)WHY Do We Use Q/F ➔ The Discriminant

- Based on the discriminant, indicate how many and what type of solutions there would be given the following equations:
- (a) $3x^2 + x + 10 = 0$
- (b) $x^2 – 8x = -16$
- (c) $3x^2 = -7x - 2$
- Verify your results using (i) an alternate algebraic method and (ii) graphically

(D)WHY Do We Use Q/F ➔ The Discriminant

- Determine the value of *W* such that $f(x) = Wx^2 + 2x – 5$ has one real root. Verify your solution (i) graphically and (ii) using an alternative algebraic method.
- Determine the value of *b* such that $f(x) = 2x^2 + bx – 8$ has no solutions. Explain the significance of your results.
- Determine the value of *b* such that $f(x) = 2x^2 + bx + 8$ has no solutions.
- Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real roots.
- Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real rational roots.

(E) Examples – Equation Writing and Forms of Quadratic Equations

- (1) Write the equation of the parabola that has zeroes of –3 and 2 and passes through the point (4,5).
- (2) Write the equation of the parabola that has a vertex at (4, −3) and passes through (2, −15).
- (3) Write the equation of the parabola that has a y – intercept of –2 and passes through the points (1, 0) and (−2,12).

(F) Homework

- HW
- Ex 8E, Q1acfghi; Q2abdef
- Ex 8H, Q5ghijkl
- Ex 8I.1, Q1bcd, Q2abc, Q3bcf
- Ex 8I.2, Q1cef, Q2ac, Q3