Lesson 5 – Algebra of Quadratic Functions - Factoring

IB Math SL1 - Santowski

Lesson Objectives

- (1) Review of Factoring trinomials
- (2) Develop the graphic significance of factors/roots
- (3) Solving Eqn (algebra/graphic connection)

BIG PICTURE

- Sometimes the same function type can be written in a **variety of different forms**. WHY?
- Is there a **connection** between the form that the **equation** is written in and some of the key features of the **graphs**????
- Since Quad. Eqns come in different forms, what are the **algebraic manipulations** that can be used to **analyze** each form of the quad eqn?

(A) Definition of Terms

- To **expand** means to write a product of expressions as a sum or difference of terms
- Ex. Expand $(m)(a + b) = (ma) + (mb)$
- Ex. Expand $(x + y)(a + b) = (xa) + (xb) + (ya) + (yb)$
- To **factor** means to write a sum or difference of terms as a product of expressions
- Ex. Factor $3x + 6 = (3)(x + 2)$
- Ex. Factor $x^2 – x – 2 = (x – 2)(x – 1)$
- The processes of expanding and factoring are REVERSE processes

(B) Factoring Methods – Common Factoring

- Some expressions can be factored by looking for a common factor ➔ usually the GCF

- (a) $2x + 8$ (b) $12x + 36$
- (c) $2x^2 + 8x$ (d) $2x^2y^2z + 8xyz^2$
- (e) $-2x – 8$ (f) $-2x + 8$
- (g) $2x – 8$ (h) $Ax + 4A$
- (i) $Ax^2 + 4Ax$ (j) $x(x - 6) + 2(x - 6)$
- (k) $y(4 - y) + 5(4 - y)$
- (l) $y(4 - y) + 5(y - 4)$

(C) Factoring Methods – Simple Trinomials

- Simple trinomials (where a = 1) are the result of the expansion of multiplying 2 binomials, so when we factor the trinomial, we are working backward to find the 2 binomials that had been multiplied to produce the trinomial in the first place
- Ex. Expand $(x + 4)(x – 2)$ ➔ $x^2 + 2x – 8$
- Ex. Factor $x^2 + 2x – 8$ ➔ $(x + 4)(x – 2)$

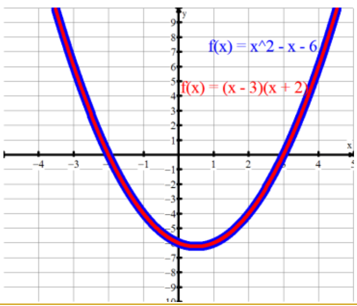
(C) Factoring Methods – Simple Trinomials

- Factor the following trinomials:
- (a) $x^2 + 5x + 6$ (b) $x^2 - x - 6$
- (c) $x^2 + 3x + 2$ (d) $c^2 + 2c – 15$
- (e) $3x^2 + 24x + 45$ (f) $2y^2 - 2y - 60$

CONTEXT for the Algebraic Skill

- So we can factor ➔ what's the point?
- Now consider the expressions as functions
- Now $x^2 - x – 6$ becomes $f(x) = x^2 - x – 6$
- Now we can graph $f(x) = x^2 - x – 6$
- Now we can graph $f(x) = (x – 3)(x + 2)$
- So we have the two forms of a quadratic equations (standard & factored) ➔ So what?

CONTEXT for the Algebraic Skill



CONTEXT for the Algebraic Skill

- So there is a connection between the algebra and the graph
- This will allow us to simply re-express an equation in standard form as an equation in factored form
- We can now SOLVE a quadratic equation in the form of $0 = x^2 - x – 6$ by FACTORING and we can solve $0 = (x – 3)(x + 2)$
- So what are we looking for graphically ➔ the roots, zeroes, x-intercepts

Zero Product Property

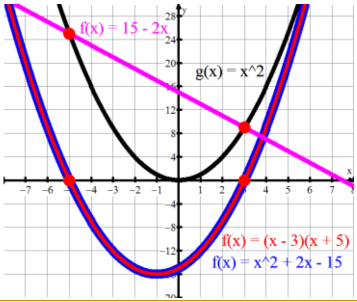
- If the product of two numbers is 0, then it must follow that ...... ???
- Mathematically, if ab = 0, then .....
- So, if $(x – r_1)(x – r_2) = 0$, then ......

CONTEXT for the Algebraic Skill

- So SOLVING by factoring is now ONE strategy for solving quadratic equations
- Solve the following equations:
- (a) $0 = x^2 + 5x + 6$ (b) $0 = x^2 - x - 6$
- (c) $x^2 + 3x = -2$ (d) $-3x^2 = 24x + 45$
- (e) $2y^2 - 2y – 60 = 0$
- (f) $x^2 = 15 – 2x$
- (g) Solve the system

$$\begin{cases} y = x^2 \\ y = 15 - 2x \end{cases}$$

CONTEXT for the Algebraic Skill



(D) Factoring Methods – Trinomials $ax^2 + bx + c$ ➔ Decomposition

- What if the leading coefficient is NOT 1?
- i.e. $3x^2 – 7x – 6$
- Consider the following EXPANSION:
- $(4x – 3)(3x + 1) = 12x^2 + 4x – 9x – 3$
- $(4x – 3)(3x +1) = 12x^2 – 5x – 3$
- Point to note is how the term -5x was “produced” ➔ from the 4x and the -9x
- NOTE the product (4x)(-9x) ➔ $-36x^2$
- NOTE the product of $(12x^2)(-3)$ ➔ $-36x^2$ ➔ (4x)(-9x)
- COINCIDENCE? I think NOT!

(D) Factoring Methods - Decomposition

- So let’s apply the observation to factor the following:
- (a) $6x^2 + 11x - 10$
- (b) $8x^2 - 18x - 5$
- (c) $9x^2 + 101x + 22$
- (d) $2x^2 + 13x + 15$
- (e) $3x^2 - 11x + 10$
- (f) $3x^2 – 7x – 6$

(E) Factoring Methods – Guess/Check

- As a valid alternative to the decomposition method, we can simply use a G/C method
- (a) $5x^2 – 17x + 6$
- (b) $6x^2 + 23x + 7$
- (c) $-36x^2 – 39x + 35$

CONTEXT for the Algebra Skill

- Find the roots of the quadratic equations:
- (a) $f(x) = 2x^2 + 13x + 15$
- (b) $f(x) = 3x^2 - 11x + 10$
- (c) $f(x) = 3x^2 – 7x – 6$
- OR
- Solve the quadratic equations:
- (d) $0 = 6x^2 + 23x + 7$
- (e) $0 = -36x^2 – 39x + 35$

CONTEXT for the Algebra Skill

- Determine the flight time of a projectile whose height, *h*(t) in meters, varies with time, *t* in seconds, as per the following formula: $h(t) = -5t^2 + 15t + 50$

(F) "Special" Factors

- The expression $a^2 - b^2$ is called a difference of squares
- Factoring a "difference of squares" produces the factors $(a + b)$ and $(a - b)$
- Factor the following:
- (a) $x^2 - 16$ (b) $4x^2 - 1$
- (c) $121 - 16x^2$ (d) $x^4 - 49$
- (e) $9x^2 - 1/9$ (e) $1/16x^2 - 3$

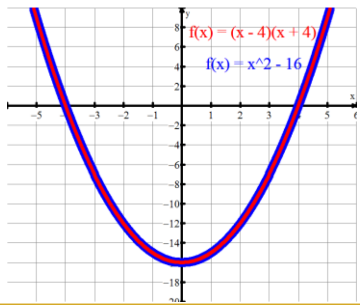
CONTEXT for the Algebra Skill

- Given these "difference of square" quadratic expressions, let's make the graphic connection
- (a) $f(x) = x^2 - 16 = (x - 4)(x + 4)$
- (b) $f(x) = 4x^2 - 1 = (2x - 1)(2x + 1)$
- (c) $f(x) = 121 - 16x^2$ (d) $f(x) = x^4 - 49$
- (e) $f(x) = 9x^2 - 1/9$ (e) $f(x) = 1/16x^2 - 3$
- So its obviously easy to find their roots, but what else do these quadratic graphs have in common?

CONTEXT for the Algebra Skill

- Given these "difference of square" quadratic expressions, let's make the graphic connection
- (a) Solve $0 = x^2 - 16$
- (b) Solve $0 = 4x^2 - 1$
- (c) Solve $0 = 121 - 16x^2$
- (d) Solve $0 = x^4 - 49$
- (e) Solve $0 = 9x^2 - 1/9$
- (f) Solve $0= 1/16x^2 - 3$

CONTEXT for the Algebra Skill



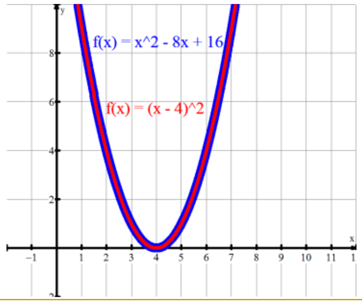
(F) "Special" Factors

- The expression $a^2 \pm 2ab + b^2$ is called a "perfect square trinomial"
- Factoring a perfect square trinomial produces the factors $(a \pm b)$ and $(a \pm b)$ which can be rewritten as $(a \pm b)^2$
- Factor the following:
- (a) $x^2 - 8x + 16$ (b) $4x^2 - 4x + 1$
- (c) $121 - 88x + 16x^2$ (d) $x^4 - 14x^2 + 49$
- (e) $9x^2 - 2x + 1/9$
- (f) Solve for b such that $1/16x^2 - bx + 3$ is a perfect square trinomial

CONTEXT for the Algebra Skill

- Given these "perfect square" quadratic expressions, let's make the graphic connection
- (a) $f(x) = x^2 - 8x + 16 = (x - 4)(x - 4) = (x - 4)^2$
- (b) $f(x) = 4x^2 - 4x + 1 = (2x - 1)(2x - 1) = (2x - 1)^2$
- (c) $f(x) = 121 - 88x + 16x^2 = (11 - 4x^2)$
- (d) $f(x) = x^4 + 14x^2 + 49 =$
- (e) $f(x) = 9x^2 + 2x + 1/9 =$
- (f) $f(x) = 1/16x^2 - bx + 3 =$
- So its obviously easy to find their roots, but what else do these quadratic graphs have in common?

CONTEXT for the Algebra Skill



CONTEXT for the Algebra Skill

- Sasha wants to build a walkway of uniform width around a rectangular flower bed that measures 20m x 30m. Her budget is \$6000 and it will cost her \$10/m² to construct the path. How wide will the walkway be?

Homework

- Ex 8D.I #1-5 odds, 6;