Lesson 59 – Derivatives of Composed Functions – The Chain Rule

IB Math SL1 - Santowski

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Fast Five

- Given the following expressions for f(x) and g(x), find the equation for fog(x)

$f(x) = x^2 \;\;\&\;\; g(x) = x^3 - x + 1$

$f(x) = \dfrac{1}{x+2} \;\;\&\;\; g(x) = \sqrt{x-3}$

$f(x) = \sin(x) \;\;\&\;\; g(x) = e^{2x}$

$f(x) = \ln\left(x^2 + \sin(x)\right) \;\;\&\;\; g(x) = \dfrac{1}{x+1}$

- Given the following composed functions, decompose them into f(x) and g(x)

$f \circ g(x) = \sqrt{x^2 - 4}$

$f \circ g(x) = \dfrac{1}{\sqrt{x}}$

$f \circ g(x) = (3x+2)^{-3}$

$f \circ g(x) = 2\sin\left(\sqrt{x^2 - 2}\right)$

$f \circ g(x) = e^{x^2 - x - 2}$

$f \circ g(x) = \ln(\tan(x))$

Lesson Objectives

- Investigate patterns in the derivatives of composed functions
- Apply the Chain Rule to differentiate composite functions
- Apply the Chain Rule in the analysis of functions
- Apply the Chain Rule in mathematical modeling

(A) Exploration

- You are given a worksheet and we will work through the first row together so that you understand what I require of you:
- Decompose $C(x) = f \circ g(x) = \left(x^2 + 4\right)^3$
- And tell me what f(x) = and what g(x) =
- Now work out the individual derivatives of f(x) and g(x)
- Now use the TI-89 (or www.calc101.com) to find the derivative of C(x) or d/dx fog(x)
- Repeat and look PATTERNS

(E) Chain Rule - Summary

- We can understand the chain rule in two notations:
- (i) $\frac{d}{dx} f \circ g(x) = f'(g(x)) \times g'(x)$
- (ii) if y is a function of u $\Rightarrow y = f(u)$:

  then $f'(x) = \frac{dy}{du} \times \frac{du}{dx}$

(F) Example

- Differentiate and simplify $h(t) = (3t^2 - 4)^{-3}$
- Differentiate and simplify $y = \frac{1}{\sqrt{4x-7}}$

(F) Example

- Differentiate and simplify $h(t) = \left(\frac{2t-1}{t+2}\right)^{-3}$
- Differentiate and do not simplify

$$y = \frac{(2x+3)^3}{\sqrt{4x-7}}$$

(G) Examples

- 1. Find the equation of the tangent line to the curve $g(x) = \frac{1}{\sqrt{20 - x^4}}$
- 2. find the location(s) of the horizontal tangent lines to the given curve

$$g(x) = (3x-8)^7 (4x+9)^5$$

(H) Examples

- (a) Where is $f(x) = (x^2 - x - 2)^5$ increasing?
- (b) Find and classify the extrema on

$$h(t) = \left(\sqrt{2t-2}\right)(2t+5)^{-1}$$

(I) Internet Links

- Calculus I (Math 2413) - Derivatives - Chain Rule from Paul Dawkins
- Visual Calculus - Chain Rule from UTK
- Applied Calculus: Everything for Calculus from Stefan Waner at Hofstra U.
- 2.3.3 Differentiation Of Compositions Of Functions - The Chain Rule From Pheng Kim Ving

(J) Homework

- Text, S4.3; pg 243-4,
- (1) algebra: Q22-41,45-50: ANV
- (2) concept: Q43-44
- (3) Applications: Q53,55,62,63,64 (MATH VIDEO)
- (4) HANDOUT (p103), Q6,11,12

(B) The Chain Rule – Function Notation

- The chain rule presents a formula that we can use to take the derivatives of these compose functions.
- (i) In function notation, we can write the chain rule as follows:
- Given that f and g are differentiable and $F = f \circ g$ is the composed function defined by $F = f(g(x))$, then $F`(x)$ is given by the product $F`(x) = f`(g(x))$ x $g`(x)$.
- We can try to understand composite functions and their derivatives in the following manner:
- $f(g(x))$ => means that f is the outer function into which we have substituted an inner function of g. So the derivative is then the product of the derivative of the outer function, f, evaluated at the inner function times the derivative of the inner function.

(C) The Chain Rule – Leibniz Notation

- (ii) In Leibniz notation, we can write the Chain rule as follows:
- If $y = f(u)$, where $u = g(x)$ and f and g are differentiable, then y is a differentiable function of x and $dy/dx = dy/du \times du/dx$
- We can try to understand the formula using this example:
- If we have the composed function $f(x) = (2x^2 + 3)^{½}$, then we could "decompose" the function into $y = f(u) = (u)^{½}$ where $u(x) = 2x^2 + 3$.
- So if we want the derivative of $f(u) = (u)^{½}$, then we can understand that the variable in $y = f(u)$ is u, so we can only take the derivative of y with respect to u, hence the idea of dy/du.
- However, we were asked for the derivative of the function F with respect to x, so we then simply "follow up" the derivative of dy/du by differentiating u with its variable of x, hence the idea of du/dx

(D) The Chain Rule

- Find dy/dx if $y = (2x^2 + 3)^{½}$
- Let $u(x) = 2x^2 + 3$ and then $y(u) = u^{½}$
- The derivative formula is $dy/dx = dy/du \times du/dx$ ➔ so …
- If $y(u) = u^{½}$, then $dy/du = ½\, u^{-½}$
- Then if $u(x) = 2x^2 + 3$, then $du/dx = 4x$ ➔ so …
- If we put it all together ➔ $dy/dx = dy/du \times du/dx$ ➔ we get
- $dy/dx = (½\, u^{-½}) \times (4x)$ and then $dy/dx = [½(2x^2 + 3)^{-½}] \times (4x)$
- So then $dy/dx = 2x(2x^2 + 3)^{-½}$