Lesson Objectives

- 1. Develop the quotient rule us
- 2. Use the quotient rule to evaluate derivatives
- 2. Apply the quotient rule to an analysis of functions
- 3. Apply the quotient rule to real world problems

Fast Five

- 1. If H = f/g, solve for f
- 2. Evaluate and interpret $\lim_{x \to 2} \frac{-3}{x-2}$
- 3. Solve for x if $x^2 - x - 6 > 0$
- 4. Solve for x if $\frac{x+1}{2x-3} > 0$
- 5. Simplify $\frac{6(x+5)^2(3x-2) - 3(3x-2)^2(x+5)}{(x+5)^6}$

(A) Derivatives of Rational Functions – The Quotient Rule

- Since the derivative of a product does not equal the product of the derivatives, what about a quotient?
- Would the derivative of a quotient equal the quotient of the derivatives?
- Since quotients are in one sense nothing more than products of a function and a reciprocal ➔ we would guess that the derivative of a quotient is not equal to the quotient of the derivatives

(A) Derivatives of Rational Functions – The Quotient Rule

- If a rational function can be written in the form $R(x)=\frac{f(x)}{g(x)}$, rewrite f(x) as ?????? And determine the expression for d/dx of R(x)

(B) Quotient Rule - Derivation

- First, set up a division and then rearrange the division to produce a multiplication so that we can apply the product rule developed earlier

$$H(x)=\frac{f(x)}{g(x)}$$
$$f(x)=H(x)\times g(x)$$
$$f'(x)=H'(x)\times g(x)+H(x)\times g'(x)$$
$$H'(x)=\frac{f'(x)-H(x)\times g'(x)}{g(x)}$$
$$H'(x)=\frac{f'(x)-\frac{f(x)}{g(x)}\times g'(x)}{g(x)}$$
$$H'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

(C) Examples Using the Quotient Law

- Differentiate each of the following rational functions. Simplify the derivative.
- ex 1. $f(x)=\frac{2x+5}{3x-1}$
- ex 2. $g(x)=\frac{x^3-3}{1+4x^2}$
- ex 3. $h(x)=\frac{x^2}{(x-2)(x^4-x^2)}$

(C) Examples Using the Quotient Law

- Determine the equation of the line normal to $g(x)=\frac{2x^2-x-3}{x^2-1}$ at x = 0

(D) Function Analysis of Rational Functions

- Make a 2 column worksheet. In the first column, briefly state what needs to be done and why and in the second column, do it!
- For what intervals is the function $f(x)=\frac{1}{x^2+1}$ concave down?

(D) Function Analysis of Rational Functions - HINTS

- Determine the domain of f(x)
- Determine the vertical asymptotes and the behaviour along the VA using limits
- Determine the horizontal asymptotes using limits
- Determine the x- and y-intercepts

(D) Function Analysis of Rational Functions

- Again, make a 2 column worksheet. In the first column, briefly state what needs to be done and why and in the second column, simply do what you said needs to be done!
- Find the intervals of increase/decrease and max/min for the given function. Then sketch the function based on your intervals. $y=\frac{3x+7}{2x+5}$

(D) Function Analysis of Rational Functions - HINTS

- Determine the domain of f(x)
- Determine the vertical asymptotes and the behaviour along the VA using limits
- Determine the horizontal asymptotes using limits
- Determine the x- and y-intercepts

(D) Function Analysis of Rational Functions

- Again, make a 2 column worksheet. In the first column, briefly state what needs to be done and why and in the second column, simply do what you said needs to be done!
- Find the intervals of increase/decrease and extrema for the given function. Sketch f(x) as well.

$$f(x) = \frac{x^2 - 8}{x - 3}$$

(E) Applications - Economics

- Suppose that the total cost in hundreds of dollars of producing x barrels of oil is given by the function $C(x) = 4x^2 + 100x + 500$. Determine the following.
- (a) the cost of producing 5000 barrels of oil
- (b) the cost of producing 5001 barrels of oil
- (c) the cost of producing the 5001st barrel of oil
- (d) C `(5000) = the marginal cost at a production level of 5000 barrels of oil. Interpret.
- (e) The production level that minimizes the average cost (where AC(x) = C(x)/x))

(E) Applications

- The number of used cars that Qassem sells per week, *t* weeks after dropping out of Calculus class, is

$$N(t) = \frac{300t^2}{1 + t^2}$$

- (a) At what rate is the number of sales changing after the 1st week? After the 5th week?
- (b) Does the number of sales per week decrease at any time during the first ten week period?

(G) Internet Links

- Calculus I (Math 2413) - Derivatives - Product and Quotient Rule
- Visual Calculus - Calculus@UTK 3.2
- solving derivatives step-by-step from Calc101

(F) Homework

- Text, S4.2, p234

- (1) Algebra: Q1-28 odds
- (2) Word Problems: Q36-47
- (3) Word problems: Q50,51,53,54,55 from pg 225

- (4) "A" Level, Worksheet (p72), Q2,3,5,7,8 & (p81), Q2,3,4