Lesson 57 – Product Rule

IB Math SL1 - Santowski

5/4/2011

1

Lesson Objectives

- 1. Develop the product rule
- 2. Apply the product rule to an analysis of functions

(A) Review

- The power rule tells us how to find the derivative of any power function $y = x^n$ which works for any real value of *n*
- The derivative of a sum/difference is simply the sum/difference of the derivatives i.e. (f + g)\` = f\` + g\`

(B) Product Rule – An Investigation

- Now the question concerns products of functions → is the derivative of a product of functions the same as the product of the derivatives? i.e. is (fg)\` = f\` x g\` ??
- Let's investigate with a product function:
- $h(x) = f(x)g(x)$ where $f(x) = x^2$ and $g(x) = x^2 - 2x$.
- Thus $h(x) = x^2(x^2 - 2x)$

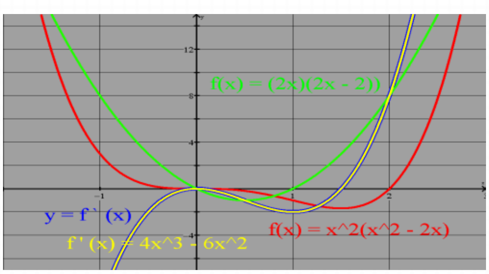
(B) Product Rule – An Investigation

- Trial 1 ➔ if we go with our idea that (fg)\` = f\` x g\` ➔ then (fg)\` = (2x)(2x - 2) = $4x^2$ - 4x
- We can graph h(x) on the GC, graph the derivative and then program in $4x^2$ - 4x and compare it to the calculated derivative from the GC.
- We will find that ➔
- And our conclusion is ➔

(B) Product Rule – An Investigation

- Trial 2 ➔ If we simply tried expanding h(x) = x^4 - $2x^3$ and then taking the derivative, we would get $4x^3$ – $6x^2$. If we program $4x^3$ – $6x^2$ into the GC, we find that we match the GC generated derivative exactly.
- So then HOW do we predict the derivative of a product IF WE CANNOT EXPAND?????

(B) Product Rule – An Investigation

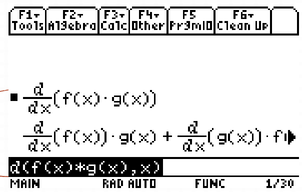


(B) Product Rule – An Investigation

- So then HOW do we predict the derivative of a product IF WE CANNOT EXPAND?????
- USE a CAS calculator like the TI-89

(C) Explore

- Try to predict the derivative of the PRODUCT of f(x)*g(x)
- Use the TI-89 to validate your prediction
- Interpret the answer from the TI-89



(D) Product Rule

- To find the derivative of a product of two functions, say $f(x)$ and $g(x)$:

$$\frac{d}{dx}\left(f(x)\times g(x)\right)=f'(x)\times g(x)+f(x)\times g'(x)$$

- To find the derivative of a product of two functions, say u and v:

$$\frac{d}{dx}(uv)=u'v+uv'$$

(E) Product Rule - Examples

- 1. Find f'(x) if f(x) = $3x^4(5x^3 + 5x - 7)$
- 2. Find the derivative of $(x^4 + x^2 - 1)(x^2 - 2)$
- 3. Differentiate $g(x)=\left(x-\frac{1}{\sqrt{x}}\right)\left(x^2+2x^{\frac{1}{3}}\right)$

(E) Product Rule - Examples

- ex 1. Find the derivative of f(x) = $3x^4(5x^3 + 5x - 7)$
- ex 2. Find the derivative of f(x) = $(x^4-4x^3-2x^2+5x+2)^2$
- ex 3. Find the equation of the tangent to the function f(x) = $(2x + 4)(3x^3 - 3x^2 + x - 2)$ at (1,-6)

(F) Examples - Applications

- Ex 1. Find the instantaneous rate of change at x = 1 of $f(x) = (x^4 - 4x^3 – 2x^2 + 5x + 2)^2$
- Ex 2. Find the equation of the tangent to $f(x) = (2x + 4)(3x^3 – 3x^2 + x - 2)$ at (1,-6)
- Ex 3. For $f(x) = (4x – 8)(2x^2 + 2x + 4)$, determine the critical numbers, the intervals of increase & decrease and then sketch f(x)

(G) Internet Links

- Calculus I (Math 2413) - Derivatives - Product and Quotient Rule
- Visual Calculus - Calculus@UTK 3.2
- solving derivatives step-by-step from Calc101

(H) Homework