Lesson 55 - Graphical Differentiation

IB Math SL1 - Santowski

4/24/2011

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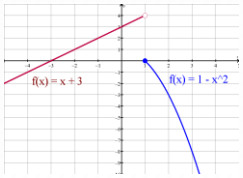
Lesson Objectives

- 1. Given the equation of a function, graph it and then make conjectures about the relationship between the derivative function and the original function
- 2. From a function, sketch its derivative
- 3. From a derivative, graph an original function

Fast Five

- 1. Find f(x) if d/dx f(x) = $-x^2 + 2x$
- 2. Sketch a graph whose first derivative is always negative
- 3. Graph the derivative of the function
- 4. If the graph represented the derivative, sketch the original function



(A) Important Terms

- **turning point**:
- **maximum**:
- **minimum**:
- **local vs absolute max/min:**
- **"end behaviour"**
- **increase:**
- **decrease:**
- **"concave up"**
- **"concave down"**

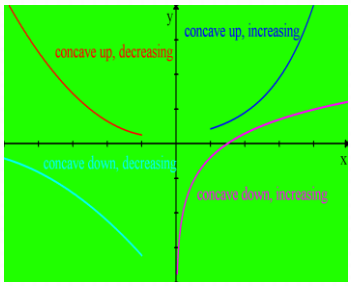
(A) Important Terms

- Recall the following terms as they were presented in previous lessons:
- **turning point**: points where the direction of the function changes
- **maximum**: the highest point on a function
- **minimum**: the lowest point on a function
- **local vs absolute**: a max can be a highest point in the entire domain (absolute) or only over a specified region within the domain (local). Likewise for a minimum.
- **"end behaviour"**: describing the function values (or appearance of the graph) as *x* values getting infinitely large positively or infinitely large negatively or approaching an asymptote

(A) Important Terms

- **increase:** the part of the domain (the interval) where the function values are getting larger as the independent variable gets higher; if f(x1) < f(x2) when x1 < x2; the graph of the function is going up to the right (or down to the left)
- **decrease:** the part of the domain (the interval) where the function values are getting smaller as the independent variable gets higher; if f(x1) > f(x2) when x1 < x2; the graph of the function is going up to the left (or down to the right)
- **"concave up"** means in simple terms that the "direction of opening" is upward or the curve is "cupped upward"
- **"concave down"** means in simple terms that the "direction of opening" is downward or the curve is "cupped downward"

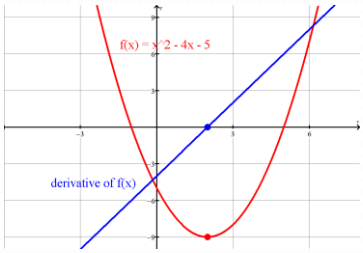
(A) Important Terms



(C) Functions and Their Derivatives

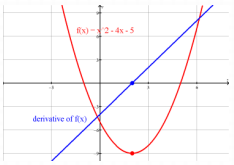
- In order to "see" the connection between a graph of a function and the graph of its derivative, we will use graphing technology to generate graphs of functions and simultaneously generate a graph of its derivative
- Then we will connect concepts like max/min, increase/decrease, concavities on the original function to what we see on the graph of its derivative

(D) Example #1



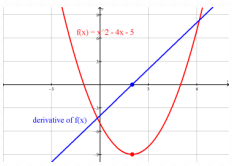
(D) Example #1

- Points to note:
- (1) the fcn has a minimum at x=2 and the derivative has an x-intercept at x=2
- (2) the fcn decreases on (-∞,2) and the derivative has negative values on (-∞,2)
- (3) the fcn increases on (2,+∞) and the derivative has positive values on (2,+∞)
- (4) the fcn changes from decrease to increase at the min while the derivative values change from negative to positive

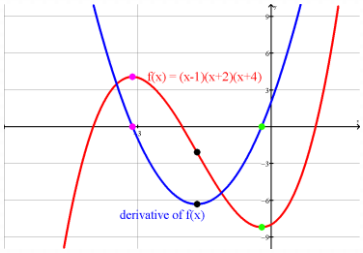


(D) Example #1

- Points to note:
- (5) the function is concave up and the derivative fcn is an increasing fcn
- (6) The second derivative of f(x) is positive

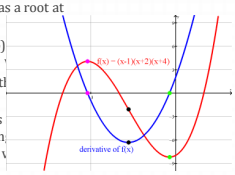


(E) Example #2



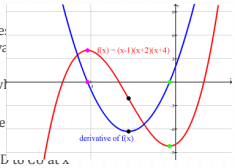
(E) Example #2

- f(x) has a max. at x = -3.1 and f`(x) has an x-intercept at x = -3.1
- f(x) has a min. at x = -0.2 and f`(x) has a root at –0.2
- f(x) increases on (-∞, -3.1) & (-0.2, ∞) the same intervals, f`(x) has positive
- f(x) decreases on (-3.1, -0.2) and on t interval, f`(x) has negative values
- At the max (x = -3.1), the fcn changes being an increasing fcn to a decreasin the derivative changes from positive negative values
- At a the min (x = -0.2), the fcn changes from decreasing to increasing ➔ the derivative changes from negative to positive



(E) Example #2

- At the max (x = -3.1), the fcn changes from being an increasing fcn to a decreasing fcn ➔ the derivative changes from positive values to negative values
- At a the min (x = -0.2), the fcn change decreasing to increasing ➔ the deriva changes from negative to positive
- f(x) is concave down on (-∞, -1.67) wh decreases on (-∞, -1.67)
- f(x) is concave up on (-1.67, ∞) while increases on (-1.67, ∞)
- The concavity of f(x) changes from CD to CU at x = -1.67, while the derivative has a min. at x = -1.67



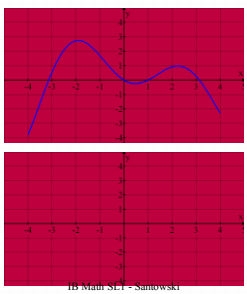
(F) Internet Links

- Watch the following animations which serve to illustrate and reinforce some of these ideas we saw in the previous slides about the relationship between the graph of a function and its derivative
- (1 Relationship between function and derivative function illustrated by IES
- (2) Moving Slope Triangle Movie

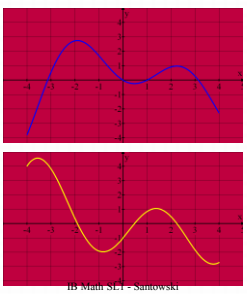
(G) Matching Function Graphs and Their Derivative Graphs

- To further visualize the relationship between the graph of a function and the graph of its derivative function, we can run through some exercises wherein we are given the graph of a function ➔ can we draw a graph of the derivative and vice versa

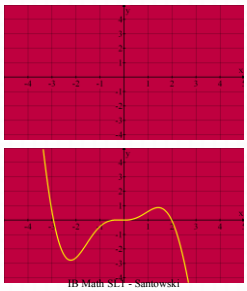
(G) Matching Function Graphs and Their Derivative Graphs



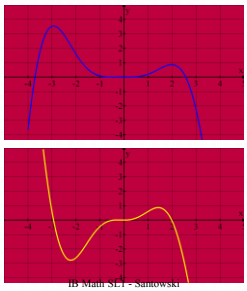
(G) Matching Function Graphs and Their Derivative Graphs - Answer



(G) Matching Function Graphs and Their Derivative Graphs – Working Backwards



(G) Matching Function Graphs and Their Derivative Graphs – Working Backwards

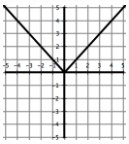


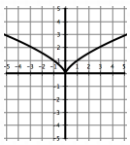
(G) Matching Function Graphs and Their Derivative Graphs - Internet Links

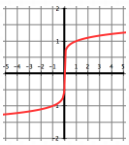
- Work through these interactive applets from maths online Gallery - Differentiation 1 wherein we are given graphs of functions and also graphs of derivatives and we are asked to match a function graph with its derivative graph
- Another exercise on sketching a derivative from an original is found here

(H) Continuity and Differentiability

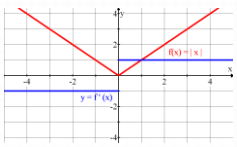
- Graph the derivatives of the following three functions:

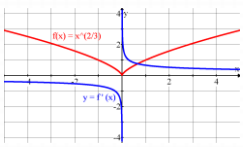


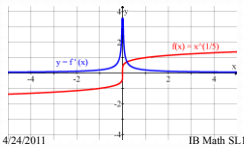




(H) Continuity and Differentiability





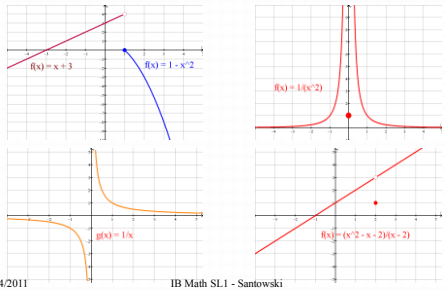


- Continuous functions are non-differentiable under the following conditions:
  - The fcn has a "corner" (ex 1)
  - The fcn has a "cusp" (ex 2)
  - The fcn has a vertical tangent (ex 3)
- This non-differentiability can be seen in that the graph of the derivative has a discontinuity in it!

(H) Continuity and Differentiability

- If a continuous function as a cusp or a corner in it, then the function is not differentiable at that point => see graphs on the next slide and decide how you would draw tangent lines (and secant lines for that matter) to the functions at the point of interest (consider drawing tangents/secants from the left side and from the right side)
- As well, included on the graphs are the graphs of the derivatives (so you can make sense of the tangent/secant lines you visualized)

(H) Continuity and Differentiability



(H) Continuity and Differentiability

- Follow this link to One-sided derivatives from IES Software
- And then follow this link to Investigating Differentiability of Piecewise Functions from D. Hill (Temple U.) and L. Roberts (Georgia College and State University

(K) Homework

- Textbook, p201-204
- Q1,2,4,6 (explanations required)
- Q7-14 (sketches required)
- Q15-19 (word problems)
- photocopy Hughes-Hallett p 115

Internet Links

- http://www.calculusapplets.com/derivfunc.html
- http://rowdy.mscd.edu/~talmanl/MTH1410U08/Pictures_080529/
- http://archives.math.utk.edu/visual.calculus/3/graphing.14/index.html
- http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/graphingdirectory/Graphing.html#PROBLEM 2
- http://www.mathdemos.org/mathdemos/derivative_sketch/sketch_the_derivative.html
- http://hypatia.math.uri.edu/~pakula/DerDraw.htm
- http://www.univie.ac.at/future.media/moe/tests/diff1/ableerkennen.html