Lesson 4 – Graphs of Quadratic Functions

IB Math SL1 - Santowski

Lesson Objectives

- (1) Establish a context for Quadratic Relations
- (2) Features of graphs of Quadratic relations ➔ D,R,intercepts, vertex (extrema/max/min), axis of symmetry, direction of opening, increase/decrease
- (3) Introduce Forms of Quad. Eqns ➔ Standard, Vertex (transformational), intercept

BIG PICTURE

- Each type of function that we will be studying in this course will have some **features common** with other types of functions BUT will also have some features **unique** to itself
- Sometimes the same function type can be written in a **variety of different forms**. WHY?
- Is there a **connection** between the form that the **equation** is written in and some of the key features of the **graphs**????

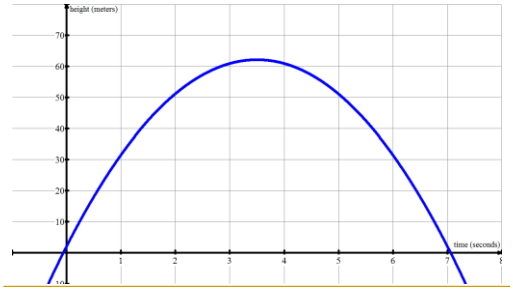
(A) Context for Quadratic Relations

- The formula for the height, *h* in meters, of an object launched into the air as a function of its time in flight, *t* in seconds, is given by is $h(t) = -\frac{1}{2} gt^2 + v_o t + h_o$
- g represents the acceleration due to gravity which is about 9.8 m/s^2, v_o refers to the launch velocity in m/s and h_o represents the initial launch height in m.
- So, if a projectile has an initial velocity of 34.3 m/s and is launched 2.1 m above the ground, determine the equation that models the flight of the projectile

(A) Context for Quadratic Relations

- If a projectile has an initial velocity of 34.3 m/s and is launched 2.1 m above the ground, graphically determine:
- (1) the time at which the projectile reaches the maximum height
- (2) the maximum height reached by the projectile
- (3) Evaluate and interpret h(2)
- (4) Solve and interpret 12 = h(t)
- (5) State the domain and range of the relation and explain WHY
- (6) The x-intercepts and their significance
- (7) The total time of flight of the projectile

(A) Context for Quadratic Relations



(B) Graphic Analysis of Parabolas

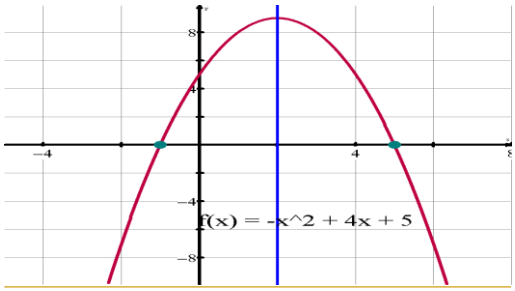
- For our investigation of quadratic functions, you will need to familiar with the following terms:

- Domain and Range
- Y-intercepts
- X-intercepts, roots, zeroes
- Vertex, maximum, minimum ➔ extrema
- Direction of opening
- Axis of symmetry
- Intervals of increase/decrease

(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations

- Graph the parabola $f(x) = -x^2 + 4x + 5$ and provide a complete graphical analysis of the parabola. Use your TI-84 to graph and analyze the parabola
- You will eventually NOT have access to a calculator to help with the functional analysis
- You will provide info about Domain, Range, Y-intercept(s), X-intercepts (AKA roots, zeroes), extrema (AKA maximum, minimum, Vertex), Direction of opening, Axis of symmetry, Intervals of increase/decrease

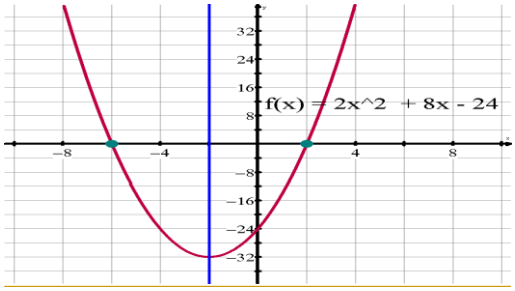
(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations



(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations

- Graph the parabola $f(x) = 2x^2 + 8x - 24$ and provide a complete graphical analysis of the parabola. Use your TI-84 to graph and analyze the parabola
- You will eventually NOT have access to a calculator to help with the functional analysis
- You will provide info about Domain, Range, Y-intercept(s), X-intercepts (AKA roots, zeroes), extrema (AKA maximum, minimum, Vertex), Direction of opening, Axis of symmetry, Intervals of increase/decrease

(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations



(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations

- Given the various features that you have seen in the graphs and listed in your analysis, is there an easy/apparent connection between the equation $f(x) = ax^2 + bx + c$ and:

- The equation and the y-intercept
- The equation and the axis of symmetry
- The eqn and intervals of inc/dec
- The equation and the vertex
- The equation and the range
- The equation and the direction of opening
- The equation and the concavity

(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations

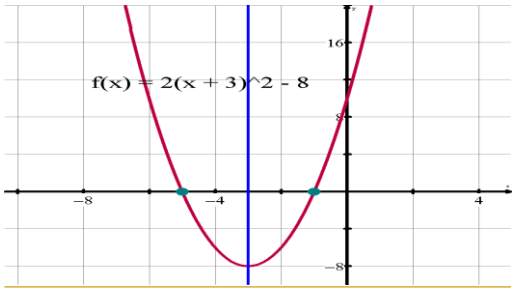
- Given the various features that you have seen in the graphs and listed in your analysis, is there an easy/apparent connection between the equation $f(x) = ax^2 + bx + c$ and:

- The equation and the y-intercept ➔ (0,c)
- The equation and the axis of symmetry ➔ (x = -b/2a)
- The eqn and intervals of inc/dec ➔ (x > -b/2a or x < -b/2a)
- The equation and the vertex ➔ (-b/2a, f(-b/2a))
- The equation and the range ➔ (y > f(-b/2a)) or y < f(-b/2a))
- The equation and the direction of opening ➔ (sign of a)
- The equation and the concavity ➔ (sign of a)

(C) Graphic Analysis of Parabolas - Vertex Form of Quadratic Equations

- Graph the parabola $f(x) = 2(x + 3)^2 - 8$ and provide a complete graphical analysis of the parabola. Use your TI-84 to graph and analyze the parabola
- You will eventually NOT have access to a calculator to help with the functional analysis
- You will provide info about Domain, Range, Y-intercept(s), X-intercepts (AKA roots, zeroes), Vertex (AKA maximum, minimum, extrema), Direction of opening, Axis of symmetry, Intervals of increase/decrease

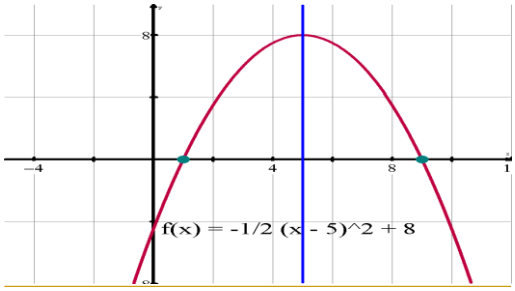
(C) Graphic Analysis of Parabolas - Vertex Form of Quadratic Equations



(C) Graphic Analysis of Parabolas - Vertex Form of Quadratic Equations

- Graph the parabola $f(x) = -\frac{1}{2}(x - 5)^2 + 8$ and provide a complete graphical analysis of the parabola. Use your TI-84 to graph and analyze the parabola
- You will eventually NOT have access to a calculator to help with the functional analysis
- You will provide info about Domain, Range, Y-intercept(s), X-intercepts (AKA roots, zeroes), Vertex (AKA maximum, minimum, extrema), Direction of opening, Axis of symmetry, Intervals of increase/decrease, Concavity

(C) Graphic Analysis of Parabolas - Vertex Form of Quadratic Equations



(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations

- Given the various features that you have seen in the graphs and listed in your analysis, is there an easy/apparent connection between the equation $f(x) = a(x - k)^2 + h$ and:

- The equation and the y-intercept
- The equation and the axis of symmetry
- The eqn and intervals of increase/decrease
- The equation and the vertex
- The equation and the range
- The equation and the direction of opening
- The equation and the concavity

(B) Graphic Analysis of Parabolas – Standard form Quadratic Equations

- Given the various features that you have seen in the graphs and listed in your analysis, is there an easy/apparent connection between the equation $f(x) = a(x – k)^2 + h$ and

- The equation and the y-intercept ➔ $(0,f(0)) = ak^2 + h)$
- The equation and the axis of symmetry ➔ $(x = k)$
- The eqn and intervals of increase/decrease ➔ $(x > k$ or $x < k)$
- The equation and the vertex ➔ $(h, f(k) = h)$
- The equation and the range ➔ $(y > h)$ or $y < h)$
- The equation and the direction of opening ➔ (sign of a)
- The equation and the concavity ➔ (sign of a)

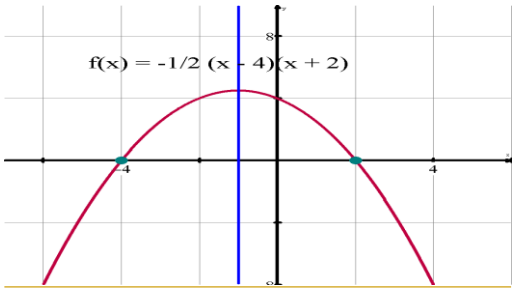
(C) Graphic Analysis of Parabolas - Factored Form of Quadratic Equations

- Graph the parabola $f(x) = - ½(x + 4)(x – 2)$ and provide a complete graphical analysis of the parabola. Use your TI-84 to graph and analyze the parabola

- You will eventually NOT have access to a calculator to help with the functional analysis

- You will provide info about Domain, Range, Y-intercept(s), X-intercepts (AKA roots, zeroes), Vertex (AKA maximum, minimum, extrema), Direction of opening, Axis of symmetry, Intervals of increase/decrease, Concavity, Continuity

(C) Graphic Analysis of Parabolas - Factored Form of Quadratic Equations



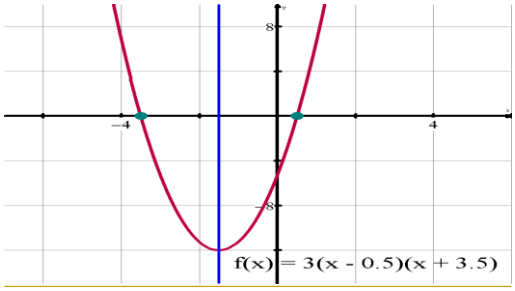
(C) Graphic Analysis of Parabolas - Factored Form of Quadratic Equations

- Graph the parabola $f(x) = 3(x – ½)(x + 3.5)$ and provide a complete graphical analysis of the parabola. Use your TI-84 to graph and analyze the parabola

- You will eventually NOT have access to a calculator to help with the functional analysis

- You will provide info about Domain, Range, Y-intercept(s), X-intercepts (AKA roots, zeroes), Vertex (AKA maximum, minimum, extrema), Direction of opening, Axis of symmetry, Intervals of increase/decrease, Concavity, Continuity

(C) Graphic Analysis of Parabolas - Factored Form of Quadratic Equations



(C) Graphic Analysis of Parabolas - Factored Form of Quadratic Equations

- Given the various features that you have seen in the graphs and listed in your analysis, is there an easy/apparent connection between the equation $f(x) = a(x – r_1)(x – r_2)$ and:

- The equation and the y-intercept
- The equation and the roots/zeroes
- The equation and the axis of symmetry
- The eqn and intervals of increase/decrease
- The equation and the vertex
- The equation and the range
- The equation and the direction of opening
- The equation and the concavity

(C) Graphic Analysis of Parabolas - Factored Form of Quadratic Equations

- Given the various features that you have seen in the graphs and listed in your analysis, is there an easy/apparent connection between the equation $f(x) = a(x - r_1)(x - r_2)$ and:

- The equation and the y-intercept ➔ $(0,f(0)) = (0,ar_1r_2)$
- The equation and the roots/zeroes ➔ $(r_1,0)$ and $(r_2,0)$
- The equation and the axis of symmetry ➔ $x = ½ (r_1+r_2)$
- The eqn and intervals of inc/dec ➔ $x < ½ (r_1+r_2)$ or $x > ½ (r_1+r_2)$
- The equation and the vertex ➔ $(½ (r_1+r_2),f(½ (r_1+r_2)))$
- The equation and the range ➔ $y >$ or $y < f(½ (r_1+r_2))$
- The equation and the direction of opening ➔ sign of a
- The equation and the concavity ➔ sign of a

(D) Matching Graphs & Equations

- Quiz Link #1 - http://quiz.econ.usyd.edu.au/mathquiz/quadratics/index.php
- Video Link #1 - http://www.youtube.com/watch?v=mDwN1SqnMRU&feature=related
- Video Link #2 - http://www.wonderhowto.com/how-to-match-quadratic-functions-318093/
- Reading Link - http://www.purplemath.com/modules/grphquad.htm

(D) Switching Forms of the Quadratic Equations

- (1) Write the equation $f(x) = 2(x + 3)^2 - 8$ in standard form
- (2) Write the equation $f(x) = - ½ (x - 5)^2 + 8$ in standard form

- (3) Write the equation $f(x) = 2(x + 3)^2 - 8$ in factored form
- (4) Write the equation $f(x) = - ½ (x - 5)^2 + 8$ in factored form

(D) Switching Forms of the Quadratic Equations

- (1) Write the equation $f(x) = - ½(x + 4)(x – 2)$ in standard form
- (2) Write the equation $3(x – ½)(x + 3.5)$ in standard form

- (3) Write the equation $f(x) = - ½(x + 4)(x – 2)$ in vertex form
- (4) Write the equation $3(x – ½)(x + 3.5)$ in vertex form

(E) Homework

- HW:

- Ex8B.1 #2,
- Ex 8B.2 #2, 3bf, 6;
- Ex 8J, #1af, 2acgh, 3bc, 4ad;
- Ex 8H #3cd, 4acei, 5chk, 6gh