



Average Rates of Change – In-Class Investigation & Assignment

PURPOSE → predict the rate at which the world population is changing in the year 2000

Consider the following data of world population over the years

Year	Population (in millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2520
1960	3020
1970	3700
1980	4450
1990	5300
1995	5670
2000	6070
2002	6200
2004	6400
2006	6550
2010	6890



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1. Prepare a scatter-plot of the data. Use $t = 0$ for 1900
2. We are working towards finding a good estimate for the rate of change of the population in 2000. So from your work in science courses like physics, you know that we can estimate the instantaneous rate of change by drawing a tangent line to the function at our point of interest and finding the slope of the tangent line.
 - (i) So on a copy of your scatter-plot, draw the curve of best fit.
 - (ii) Then draw a tangent line at $t = 100$ (for 2000) and estimate the instantaneous rate of change of population in 2000.
 - (iii) How confident are you about your prediction.
 - (iv) Give reasons for your confidence (or lack of confidence).

To come up with a prediction for the instantaneous rate of change that we can be confident about, we will develop an algebraic method of determining a tangent slope. So work through the following exercise (Q3 – Q9) We will start by finding average rates of change, which we will use as a basis for an estimate of the instantaneous rate of change.

3. We will find the average rate of change of the population between 1950 and 2000. Mark both points and draw the secant line and then find the average rate of change.
4. Now find the average rate of change for the population between (i) 1970 and 2000, (ii) 1980 and 2000, (iii) 1990 and 2000, (iv) 1995 and 2000. Draw each secant line on your scatter plot
5. Now using the work from Question 4 & 5, we can make a prediction or an estimate for the instantaneous rate of change in 2000. (i.e at what rate is the population changing 2000)
6. Unfortunately, we have discrete data in our example, which limits us from presenting a more accurate estimate for the instantaneous rate of change. If we could generate an equation for the data, we may interpolate some data points, which we could use to prepare a better series of average rates of change so that we could estimate an instantaneous rate of change. So now find the best regression equation for the data using technology. Justify your choice of algebraic model for the population.
7. Now using our equation, we can generate interpolated values for years closer to 2000 (1996, 1997, 1998, 1999). Now determine the average rates of change between (i) 1996 and 2000, (ii) 1997 and 2000 etc...

We now have a better list of average rates of change so that we could estimate an instantaneous rate of change.

8. **Finally, what is the best estimate for the instantaneous rate of change in 2000?** How could you use the same process as in Questions 6 & 7 to get an even more accurate estimate of the instantaneous rate of change?



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9. One other option to explore: We will find the average rate of change of the population between 2010 and 2000. Mark both points and draw the secant line and then find the average rate of change.
10. Using our equation, generate other interpolated values for populations higher than 2000 (2008, 2006, 2004, 2002). Then calculate average rates of change between (i) 2008 and 2010, (ii) 2006 and 2000, etc.... which will provide another list of average rates of change.
11. **Provide another estimate for an instantaneous rate of change in 2000.**
12. Explain how this process in Question 9 & 10 is different than the option we just finished in Question 6 & 7? How is the process the same?
13. Another option to explore is as follows:
- (i) What was the average rate of change between 1990 and 2000 (see work in Question 3)?
 - (ii) What was the average rate of change between 2010 and 2000 (see work in Question 9)?
 - (iii) Average these two rates. Compare this answer to your estimate from Question 8 and 11.
 - (iv) What was the average rate of change between 1990 and 2010? Compare this value to our estimate from Question 13(iii) and from Question 8 and from 11.
 - (v) Now repeat the process from Question 9(iv) for the following:
 - (a) 1996 and 2004
 - (b) 1998 and 2002
 - (c) 1999 and 2001
 - (vi) Explain the rational (reason, logic) behind the process in this third option .
14. From your work in Questions 3 through 13:
- (i) compare and contrast the processes of manually estimating a tangent slope by drawing a tangent line and using an algebraic approach.
 - (ii) Explain the meaning of the following mathematical statement: $m_{\text{tangent}} = \lim_{x_1 \rightarrow x_2} \frac{f(x_1) - f(x_2)}{x_1 - x_2}$

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