Lesson 46 – Trigonometric Functions

IB Math SL1 - Santowski

Lesson Objectives

- Make the connection between angles in standard position and sinusoidal functions
- Graph and analyze a periodic function
- Introduce transformations of periodic functions

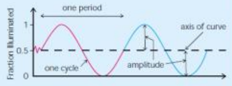
(A) Key Terms Related to Periodic Functions

- Define the following key terms that relate to trigonometric functions:
- (a) period
- (b) amplitude
- (c) axis of the curve (or equilibrium axis)

(A) Key Terms

Key Ideas

- Repeating data forms a periodic function.
- A periodic function has a self-repeating graph.
- The **cycle** of a graph is the smallest complete repeating pattern of the graph.



- The length of one cycle is called the **period**.
- The horizontal line that is halfway between the maximum and minimum values of a periodic curve is called the **axis of the curve**.
- The equation of the axis of the curve is

$$y = \frac{\text{maximum value} + \text{minimum value}}{2}$$

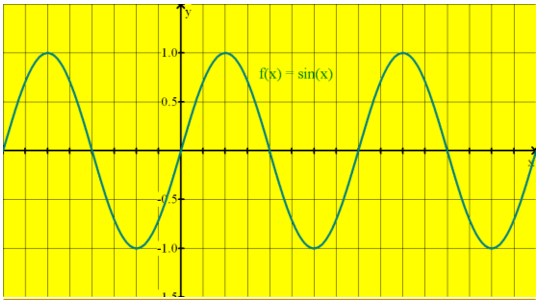
- The magnitude of the vertical distance from the axis of the curve to either the maximum or minimum value is called the **amplitude** of the function. The amplitude, a, is calculated as

$$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$

(A) Graph of f(x) = sin(x)

- We can use our knowledge of angles on Cartesian plane and our knowledge of the trig ratios of special angles to create a list of points to generate a graph of f(x) = sin(x)
- See link at http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan

(A) Graph of f(x) = sin(x)



(A) Features of f(x) = sin(x)

- The graph is **periodic** (**meaning that it repeats itself)**
- **Domain**:
- **Range**:
- **Period**: length of one cycle, how long does the pattern take before it repeats itself ➔.
- **x-intercepts**:
- **Axis of the curve** or **equilibrium axis**: ➔
- **amplitude**: max height above equilibrium position - how high or low do you get ➔
- **y-intercept**:
- **max. points**:
- **min. points:**

(A) Features of f(x) = sin(x)

- The graph is **periodic** (**meaning that it repeats itself)**
- **Domain**: x E **R**
- **Range**: [-1,1]
- **Period**: length of one cycle, how long does the pattern take before it repeats itself ➔ 360° or 2 π rad.
- **x-intercepts**: every 180°, x = 180°n where n E I or πn where n E I.
- **Axis of the curve** or **equilibrium axis**: x-axis
- **amplitude**: max height above equilibrium position - how high or low do you get => 1 unit
- **y-intercept**: (0°,0)
- **max. points**: 90°+ 360°n (or 2π + 2 π n)
- **min. points:** 270°+ 360°n or -90° + 360°n or -π/2 + 2 π n

(A) Features of f(x) = sin(x)

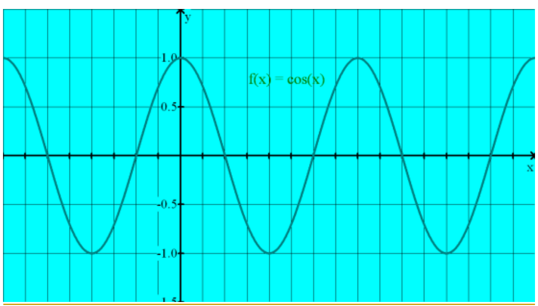
- Five point summary of f(x) = sin(x)

x					
y=f(x)					

(B) Graph of f(x) = cos(x)

- We can use our knowledge of angles on Cartesian plane and our knowledge of the trig ratios of special angles to create a list of points to generate a graph of f(x) = cos(x)
- See link at http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan

(B) Graph of f(x) = cos(x)



(B) Features of f(x) = cos(x)

- The graph is **periodic**
- **Domain**:
- **Range**:
- **Period**: length of one cycle, how long does the pattern take before it repeats itself ➔.
- **x-intercepts**:
- **Axis of the curve** or **equilibrium axis**:➔
- **amplitude**: max height above equilibrium position - how high or low do you get ➔
- **y-intercept**:
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- **min. points:**

(B) Features of f(x) = cos(x)

- The graph is **periodic**
- **Domain**: x E **R**
- **Range**: [-1,1]
- **Period**: length of one cycle, how long does the pattern take before it repeats itself ➔ 360° or 2 π rad.
- **x-intercepts**: every 180° starting at 90°, x = 90° + 180°n where n E I (or π/2 + π n where n E I)
- **Axis of the curve** or **equilibrium axis**: ➔ x-axis
- **amplitude**: max height above equilibrium position - how high or low do you get => 1 unit
- **y-intercept**: (0°,1)
- **max. points**: 0° + 360°n (2 π n)
- **min. points:** 180° + 360°n or -180° + 360°n (or π + 2 π n)

(B) Features of f(x) = cos(x)

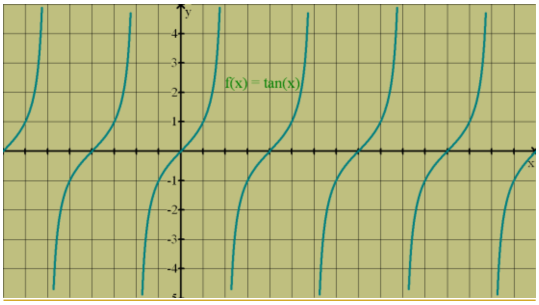
- Five point summary of f(x) = cos(x)

x					
y=f(x)					

(C) Graph of f(x) = tan(x)

- We can use our knowledge of angles on Cartesian plane and our knowledge of the trig ratios of special angles to create a list of points to generate a graph of f(x) = tan(x)
- See link at http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html#sincostan

(C) Graph of f(x) = tan(x)



(C) Features of f(x) = tan(x)

- The graph is **periodic**
- **Domain**:
- **Asymptotes**:
- **Range**:
- **Period**: length of one cycle, how long does the pattern take before it repeats itself ➔
- **x-intercepts**:
- **amplitude**: max height above equilibrium position - how high or low do you get ➔
- **y-intercept**:
- **max. points**:
- **min. points**:

(C) Features of f(x) = tan(x)

- The graph is **periodic**
- **Domain**: x E **R** where x cannot equal 90°, 270°, 450°, or basically 90° + 180°n where n E I
- **Asymptotes**: every 180° starting at 90°
- **Range**: x E **R**
- **Period**: length of one cycle, how long does the pattern take before it repeats itself = 180° or π rad.
- **x-intercepts**: x = 0°, 180°, 360°, or basically 180°n where n E I or x = πn
- **amplitude**: max height above equilibrium position - how high or low do you get => none as it stretches on infinitely
- **y-intercept**: (0°,0)
- **max. points**: none
- **min. points:** none

(C) Features of f(x) = tan(x)

- Five point summary of f(x) = tan(x)

x					
y=f(x)					

(D) Internet Links

- Unit Circle and Trigonometric Functions sin(x), cos(x), tan(x) from AnalyzeMath
- Relating the unit circle with the graphs of sin, cos, tan from Maths Online

(E) Transformed Sinusoidal Curves

- Since we are dealing with general sinusoidal curves, the basic equation of all our curves should involve f(x) = sin(x) or f(x) = cos(x)
- In our questions, though, we are considering TRANSFORMED sinusoidal functions however ➔ HOW do we know that????
- So our general formula in each case should run something along the lines of f(x) = ***a***sin(***k***(x+***c***)) + ***d***

The General Sinusoidal Equation

- In the equation f(x) = ***a***sin(***k***(x+***c***)) + ***d***, explain what:
- a represents?
- k represents?
- c represents?
- d represents?

The General Sinusoidal Equation

- In the equation f(x) = ***a***sin(***k***(x+***c***)) + ***d***, explain what:
- **a** represents? ➔ vertical stretch/compression ➔ so changes in the **amplitude**
- **k** represents? ➔ horizontal stretch/compression ➔ so changes in the **period**
- **c** represents? ➔ horizontal translations ➔ so changes in the **starting point of a cycle** (phase shift)
- **d** represents?➔ vertical translations ➔ so changes in the **axis of the curve (equilibrium)**

(D) Transforming y = sin(x)

- Graph y = sin(x) as our reference curve
- (i) Graph y = sin(x) + 2 and y = sin(x) – 1 and analyze ➔ what features change and what don't?
- (ii) Graph y = 3sin(x) and y = ¼sin(x) and analyze ➔ what features change and what don't?
- (iii) Graph y = sin(2x) and y = sin(½x) and analyze ➔ what features change and what don't?
- (iv) Graph y = sin(x+π/4) and y = sin(x-π/3) and analyze ➔ what changes and what doesn't?
- We could repeat the same analysis with either y = cos(x) or y = tan(x)

(E) Combining Transformations

- We continue our investigation by graphing some other functions in which we have combined our transformations
- (i) Graph and analyze y = 2 cos (2x) – 3 ➔ identify transformations and state how the key features have changed
- (ii) Graph and analyze y = tan(½ x + π/4) ➔ identify transformations and state how the key features have changed

(B) Writing Sinusoidal Equations

- ex 1. Given the equation y = 2sin3(x - 60°) + 1, determine the new amplitude, period, phase shift and equation of the axis of the curve.
- Amplitude is obviously 2
- Period is 2π/3 or 360°/3 = 120°
- The equation of the equilibrium axis is y = 1
- The phase shift is 60° to the right

(B) Writing Sinusoidal Equations

- ex 2. Given a cosine curve with an amplitude of 2, a period of 180°, an equilibrium axis at y = -3 and a phase shift of 45° right, write its equation.
- So the equation is y = 2 cos [2(x - 45°)] – 3
- Recall that the k value is determined by the equation period = 2π/k or k = 2π/period
- If working in degrees, the equation is modified to period = 360°/k or k = 360°/period

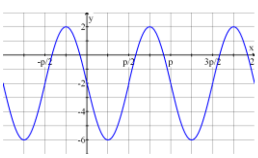
(B) Writing Sinusoidal Equations

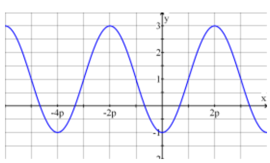
- ex 3. Write an equation and then graph each curve from the info on the table below:

	A	Period	PS	Equil
Sin	7	3π	¼ π right	-6
Cos	8	180°	None	+2
Sin	1	720°	180° right	+3
Cos	10	½ π	π left	none

(B) Writing Sinusoidal Equations

- ex 4. Given several curves, repeat the same exercise of equation writing ➔ write both a sine and a cosine equation for each graph





(E) Homework

- Nelson text,
- Section 5.2, p420, Q1-6eol, 11,12, 15-19
- Section 5.6, p455, Q1-10eol,11,13,18
- Nelson text, page 464, Q8,9,10,12,13-19