Lesson 41 - Trigonometric Equations

IB Math SL - Santowski

FAST FIVE

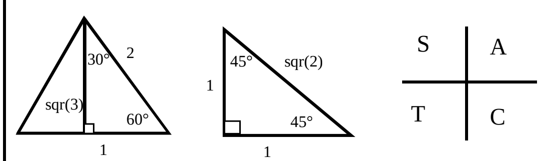
- EXPLAIN the difference between the following 2 equations:
- (a) sin(x) = 0.75
- (b) sin(0.75) = x
- Now, use you calculator to solve for *x* in both equations
- Define “principle angle” and “related acute angle”

(A) Review

- We have two key triangles to work with in terms of determining our related acute angles and we can place a related acute angle into any quadrant and then use the CAST “rule” to determine the sign on the trigonometric ratio
- The key first quadrant angles we know how to work with are 0°, 30°, 45°, 60°, and 90°

(A) Review

- The two triangles and the CAST “rule” are as follows:



(A) Review

- We can set up a table to review the key first quadrant ratios:

θ	Sin(θ)	Cos(θ)	Tan(θ)
0			
30° or π/6			
45° or π/4			
60° or π/3			
90° or π/2			

(A) Review

- We can set up a table to review the key first quadrant ratios:

θ	Sin(θ)	Cos(θ)	Tan(θ)
0	0	1	0
30° or π/6	½	√3/2	1/√3
45° or π/4	1/√2	1/√2	1
60° or π/3	√3/2	½	√3
90° or π/2	1	0	Undef.

(B) Solving Linear Trigonometric Equations

- We will outline a process by which we come up with the solution to a trigonometric equation ➔ it is important you understand WHY we carry out these steps, rather than simply memorizing them and simply repeating them on a test of quiz

(B) Solving Linear Trigonometric Equations

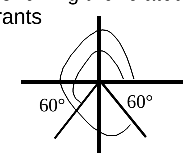
- Work with the example of sin(θ) = -√3/2
- Step 1: determine the related acute angle (RAA) from your knowledge of the two triangles
- Step 2: consider the sign on the ratio (-ve in this case) and so therefore decide in what quadrant(s) the angle must lie
- Step 3: draw a diagram showing the related acute in the appropriate quadrants
- Step 4: from the diagram, determine the principle angles

(B) Solving Linear Trigonometric Equations - Solns

- Work with the example of sin(θ) = -√3/2
- Step 1: determine the related acute angle (RAA) from your knowledge of the two triangles (in this case, simply work with the ratio of √3/2) ➔ θ = 60° or π/3
- Step 2: consider the sign on the ratio (-ve in this case) and so therefore decide in what quadrant the angle must lie ➔ quad. III or IV in this example

(B) Solving Linear Trigonometric Equations

- Step 3: draw a diagram showing the related acute in the appropriate quadrants



- Step 4: from the diagram determine the principle angles ➔ 240° and 300° or 4π/3 and 5π/3 rad.

(B) Solving Linear Trigonometric Equations

- One important point to realize ➔ I can present the same original equation (sin(θ) = - √3/2) in a variety of ways:
- (i) 2sin(θ) = - √3
- (ii) 2sin(θ) + √3 = 0
- (iii) θ = $\sin^{-1}$(- √3/2)

(C) Further Examples

- Solve the following without a calculator

$$2\cos(\theta)+2=3 \quad \text{for } \theta\in(0,4\pi)$$

$$\sin(2\theta)-0.5=0 \quad \text{for } \theta\in(0,2\pi)$$

$$\cos\left(\theta-\frac{\pi}{4}\right)+1=2 \quad \text{for } \theta\in(-2\pi,2\pi)$$

(C) Further Practice

- Solve the following for θ:

$$\sin\theta = 0 \quad \text{for } 0 \le \theta \le 4\pi$$
$$\sin\theta = 1 \quad \text{for } -2\pi \le \theta \le 2\pi$$
$$1 + \cos\theta = 0 \quad \text{for } -\pi \le \theta \le 3\pi$$
$$\tan\theta = 0 \quad \text{for } 0 \le \theta \le 3\pi$$

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(C) Further Practice

- Solve without a calculator

$$\sqrt{3} + 3\sin x = 5\sin x \quad \text{for } x \in (0,4\pi)$$
$$8\cos x + 1 = 2\cos x + 4 \quad \text{for } x \in (-4\pi,0)$$
$$\sin x - 4 = -2\sin x \quad \text{for } x \in (-2\pi,2\pi)$$
$$\sin^2 x - 3 = -3\sin^2 x \quad \text{for } x \in (0,2\pi)$$

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Review – Graphic Solutions

- We know what the graphs of the trigonometric functions look like
- We know that when we algebraically solve an equation in the form of f(x) = 0, then we are trying to find the roots/zeroes/x-intercepts
- So we should be able to solve trig equations by graphing them and finding the x-intercepts/intersection points

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(D) Modeling Periodic Phenomenon & Trig Equations

The height, h, of a basket on a water wheel at time t is given by $h(t) = \sin(6t)^\circ$, where t is in seconds and h is in metres.

(a) How high is the basket at 14 s?

(b) When will the basket first be 0.5 m under water?

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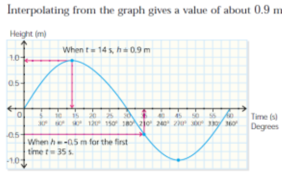
(D) Modeling Periodic Phenomenon & Trig Equations

(a) The values of h and t can be interpolated from a graph. Prepare a table. In this case, 5-s intervals were used, although the interval size could be different.

t (seconds)	0	5	10	15	20	25	30	35	40	45	50	55	60
$(6t)^\circ$	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$h(t) = \sin(6t)^\circ$ (metres)	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0

Interpolating from the graph gives a value of about 0.9 m, as shown.



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(D) Modeling Periodic Phenomenon & Trig Equations

The height, h, of a basket on a water wheel at time t is given by $h(t) = \sin(6t)^\circ$, where t is in seconds and h is in metres.

(a) How high is the basket at 14 s?

(b) When will the basket first be 0.5 m under water?

The question could also be answered by substituting into the equation.

Then,

$$\begin{aligned} h(14) &= \sin(6 \times 14)^\circ \\ &= \sin 84^\circ \\ &\doteq 0.995 \end{aligned}$$

Substitution gives a more accurate answer in this case. At 14 s, the height is almost 1 m.

A height of 0.5 m under water corresponds to a height of −0.5 m in the model. Therefore,

$$h(t) = \sin(6t)^\circ$$
$$-0.5 = \sin(6t)^\circ \qquad \text{Interpolation shows that } \sin 210^\circ = -0.5.$$

The basket will be 0.5 m under water for the first time at 35 s.

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(E) Examples (with Technology)

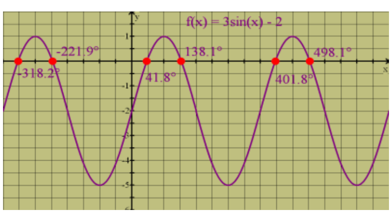
- Solve the equation 3sin(x) – 2 = 0

(E) Examples

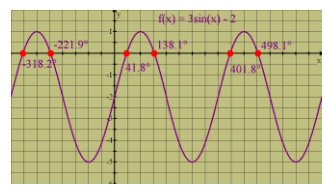
- Solve the equation 3sin(x) – 2 = 0
- The algebraic solution would be as follows:
- We can set it up as sin(x) = 2/3 so x = $\sin^{-1}(2/3)$ giving us 41.8° (and the second angle being 180° - 41.8° = 138.2°
- Note that the ratio 2/3 is not one of our standard ratios corresponding to our "standard" angles (30,45,60), so we would use a calculator to actually find the related acute angle of 41.8°

(E) Examples

- We can now solve the equation 3sin(x) – 2 = 0 by graphing f(x) = 3sin(x) – 2 and looking for the x-intercepts



(E) Examples



- Notice that there are 2 solutions within the limited domain of 0° ≤ θ ≤ 360°
- However, if we expand our domain, then we get two new solutions for every additional period we add
- The new solutions are related to the original solutions, as they represent the positive and negative co-terminal angles
- We can determine their values by simply adding or subtracting multiples of 360° (the period of the given function)

(E) Examples

- Solve the following equations:

$$6\sin\theta + 4 = 0 \quad \text{for } -\pi \le \theta \le \pi$$

$$4\sin 2\theta = 7\cos\theta \quad \text{for } -1.5 \le \theta \le 3$$

$$2\sin x - 4\sin^2 x = 2\tan\left(\frac{x}{2}\right) \quad \text{for } -8 \le \theta \le 0$$

(F) Solving Equations with Technology

- The monthly sales of lawn equipment can be modelled by the following function, where S is the monthly sales in thousands of units and t is the time in months, t = 1 corresponds to January.

$$S(t) = 32.4\sin\left(\frac{\pi}{6}\right)t + 53.5$$

- (a) How many units will be sold in August?
- (b) In which month will 70 000 units be sold?
- (c) According to this model, how many times will the company sell 70 000 units over the next ten years?

(C) Internet Links

- Introductory Exercises from U. of Sask EMR ➔ try introductory questions first, but skip those involving proving identities
- Solving Trigonometric Equations - on-line math lesson from MathTV
- Trigonometric Equations and The Unit Circle from AnalyzeMath

(D) Homework

- HH Textbook
- 13F2, Q1abcdgi
- 13F3, 2aefghi, 4ab
- 13H, 3abcd, 4acdefg