Lesson 39 – The Unit Circle

IB Math SL1 - Santowski

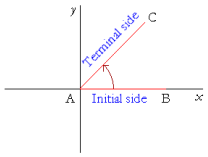
Review – Where We've Been

- We have a new understanding of angles as we have now placed angles in a circle on a coordinate plane
- We have a new unit we can use to measure angles
- We can measure the length of an arc and the area of a sector

Lesson Objectives

- Review/introduce the key ideas that result from understanding angles in standard position
- Know the trig ratios of all multiples of 30°, 45°, 60°, 90° angles
- Understand the concepts behind the trig ratios of special angles in all four quadrants
- Solve simple trig equations involving special trig ratios
- Tabulate the trig ratios to begin graphing trig functions

(A) Angles in Standard Position

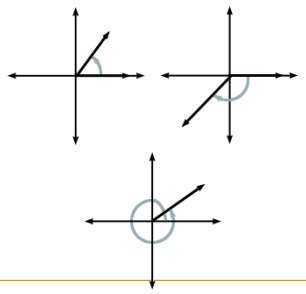


- Angles in standard position are defined as angles drawn in the Cartesian plane where the initial arm of the angle is on the *x* axis, the vertex is on the origin and the terminal arm is somewhere in one of the four quadrants on the Cartesian plane
- To form angles of various measure, the terminal arm is simply rotated through a given angle

(B) Terms Related to Standard Angles

- A **principle angle** is any angle between 0° and 360°
- A **coterminal angle** is one which shares the same terminal arm and the same initial arm as a principle angle, but was formed by several rotations of the terminal arm, so that it winds up in the same position as the terminal arm of its principle angle. Draw an example
- A **negative angle** is one which is formed from a rotation in a clockwise direction. Draw an example
- a **related acute angle** is the angle between the *x* axis and the terminal arm and will always be between 0° and 90°. Draw an example

(B) Terms Related to Standard Angles



(C) Angles in Standard Position – Interactive Applet

- Go to the link below and work through the ideas presented so far with respect to angles in standard position
- Angles In Trigonometry from AnalyzeMath

(D) Examples

- ex 1. Draw a 225° angle and label the principle angle and the related acute angle and draw one coterminal angle.
- ex 2. Determine and draw the next two consecutive positive coterminal angles and the first negative coterminal angle with 43°
- ex 3. Draw a –225° and label the principle angle and the related acute angle and draw one coterminal angle

(E) Trig Ratios of Angles in Standard Position

- We can once again set up our angle in the Cartesian plane and now simply determine the sin, cos, and tan ratios of these angles as we had in our previous lessons:
- We simply place a point on the terminal arm, determine its x,y coordinates and then drop a perpendicular from the point down to the x axis. So now we have our right triangle.
- As such, we can now define the primary trig ratios as follows:
- sine A = y/r
- cosine A = x/r
- tangent A = y/x

(F) Examples

- we will move the point A(3,4) through the four quadrants and determine the sine, cosine and tangent ratios in each of the four quadrants:
- Quadrant I - P(3,4) → sin A = 4/5, cos A = 3/5, tan A = 4/3
- Quadrant II - P(-3,4) → sin A = 4/5, cos A = -3/5, tan A = -4/3
- Quadrant III - P(-3,-4) → sin A = -4/5, cos A = -3/5, tan A = 4/3
- Quadrant IV - P(3,-4) → sin A = -4/5, cos A = 3/5, tan A = -4/3

(I) Examples

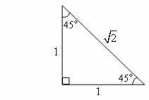
- Ex 1. The terminal arm of an angle goes through the point (-3,5).
- (i) draw a diagram showing the angle,
- (ii) determine the angle's three primary trig ratios,
- (iii) illustrate the related acute angle
- (iv) determine the angle that corresponds to each of the primary ratios. Interpret.
- Ex 2. The cosine ratio of an angle is –4/7. Draw the angle in standard position and determine the other trig ratios for the angle. What is the measure of the angle? Include a diagram

(A) Review – Special Triangles

- Review 45°- 45°- 90° triangle
- sin(45°) =
- cos(45°) =
- tan(45°) =
- csc(45°) =
- sec(45°) =
- cot(45°) =

(A) Review – Special Triangles

- Review 45-45-90 triangle



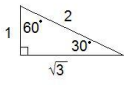
	45°
sin	$\frac{1}{\sqrt{2}}$
cos	$\frac{1}{\sqrt{2}}$
tan	1

(A) Review – Special Triangles

- Review 30°- 60°- 90° triangle ➔ 30°
  - sin(30°) =
  - cos(30°) =
  - tan(30°) =
  - csc(30°) =
  - sec(30°) =
  - cot(30°) =
- Review 30°- 60°- 90° triangle ➔ 60°
  - sin(60°) =
  - cos(60°) =
  - tan(60°) =
  - csc(60°) =
  - sec(60°) =
  - cot(60°) =

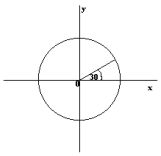
(A) Review – Special Triangles

- 30-60-90 triangle

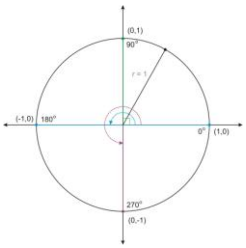


	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

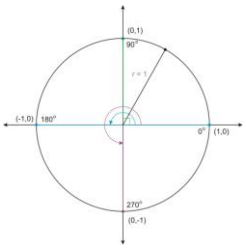
(B) Trig Ratios of First Quadrant Angles

- We have already reviewed first quadrant angles in that we have discussed the sine and cosine (as well as other ratios) of 30°, 45°, and 60° angles
- What about the quadrantal angles of 0 ° and 90°?
- 

(B) Trig Ratios of First Quadrant Angles – Quadrantal Angles

- Let's go back to the x,y,r definitions of sine and cosine ratios and use ordered pairs of angles whose terminal arms lie on the positive x axis (0° angle) and the positive y axis (90° angle)
  - sin(0°) =
  - cos (0°) =
  - tan(0°) =
  - sin(90°) =
  - cos(90°) =
  - tan(90°) =
- 

(B) Trig Ratios of First Quadrant Angles – Quadrantal Angles

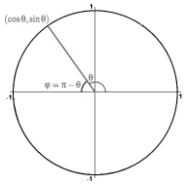
- Let's go back to the x,y,r definitions of sine and cosine ratios and use ordered pairs of angles whose terminal arms lie on the positive x axis (0° angle) and the positive y axis (90° angle)
  - sin(0°) = 0/1 = 0
  - cos (0°) = 1/1 = 1
  - tan(0°) = 0/1 = 0
  - sin(90°) = 1/1 = 1
  - cos(90°) = 0/1 = 0
  - tan(90°) = 1/0 = undefined
- 

(B) Trig Ratios of First Quadrant Angles - Summary

	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$

(C) Trig Ratios of Second Quadrant Angles

- Now let's apply the same ideas & concepts to considering special second quadrant angles of 120°, 135°, 150° and 180°



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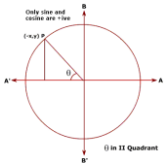
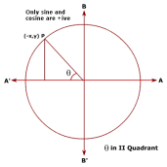


θ	Sin(θ)	Cos(θ)	Tan(θ)
120°			
150°			

(C) Trig Ratios of Second Quadrant Angles

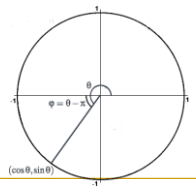
- Now let's apply the same ideas & concepts to considering special second quadrant angles of 120°, 135°, 150° and 180°



θ	Sin(θ)	Cos(θ)	Tan(θ)
135°			
180°			

(D) Trig Ratios of Third Quadrant Angles

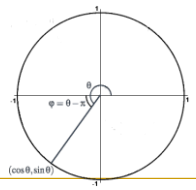
- Now let's apply the same ideas & concepts to considering special second quadrant angles of 210°, 225°, 240° and 270°



θ	Sin(θ)	Cos(θ)	Tan(θ)
210°			
240°			

(D) Trig Ratios of Third Quadrant Angles

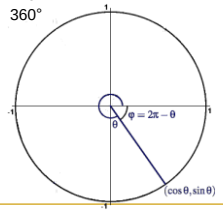
- Now let's apply the same ideas & concepts to considering special second quadrant angles of 210°, 225°, 240° and 270°



θ	Sin(θ)	Cos(θ)	Tan(θ)
225°			
270°			

(D) Trig Ratios of Third Quadrant Angles

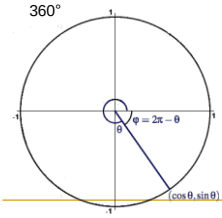
- Now let's apply the same ideas & concepts to considering special second quadrant angles of 300°, 315°, 330° and 360°



θ	Sin(θ)	Cos(θ)	Tan(θ)
300°			
330°			

(D) Trig Ratios of Fourth Quadrant Angles

- Now let's apply the same ideas & concepts to considering special second quadrant angles of 300°, 315°, 330° and 360°



θ	Sin(θ)	Cos(θ)	Tan(θ)
315°			
360°			

(G) Summary (As a Table of Values)

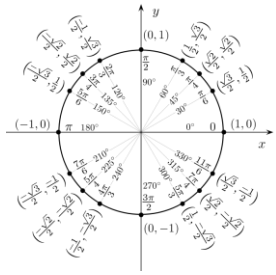
	0	30	45	60	90	120	135	150	180
sin									
cos									

	210	225	240	270	300	315	330	360
sin								
cos								

(G) Summary – As a "Unit Circle"

- The Unit Circle is a tool used in understanding sines and cosines of angles found in right triangles.
- It is so named because its radius is exactly one unit in length, usually just called "one".
- The circle's center is at the origin, and its circumference comprises the set of all points that are exactly one unit from the origin while lying in the plane.

(G) Summary – As a "Unit Circle"



Unit Circle – Interactive Applet

- Go to the link below and work through the ideas presented so far with respect to the unit circle
- Unit Circle from Trigonometry from AnalyzeMath
- http://www.mccsc.edu/~aterwill/geogebra/unitcircleapplet/Trig_Ratios_applet.html
- http://www.univie.ac.at/future.media/moe/galerie/fun2/fun2.html

(H) Examples

- Complete the worksheet:
- http://www.edhelper.com/math/trigonometry104.htm
- http://www.edhelper.com/math/trigonometry108.htm

(H) Trig Equations

- Simplify or solve

$(a)\ \sin 30° \cos 30° - \tan 30°$

$(b)\ \sin 45° \sin 30° - (\tan 60°)^2$

$(c)\ \dfrac{\sin 150°}{\sec 210°} - \csc(-330°)$

$(b)\ \sin(\theta) = -\dfrac{1}{2}$

$(c)\ 2\cos(\theta) = 1$

$(d)\ \sqrt{3}\tan(\theta) = 1$

(I) Trig Functions

- Given the table of values you have prepared previously, graph the points.

(J) Internet Links

- http://www.humboldt.edu/~dlj1/PreCalculus/Images/UnitCircle.html
- http://www.snow.edu/jonathanb/Courses/Math1060/unit_circ_trig.html
- http://www.youtube.com/watch?v=6Qv_bPlQS8E

Homework

- HW
- S13.3, p848, Q8-19, 21,23,29,31,37-48
- S13.4, p854, Q36-51