LESSON 38 – CIRCLE GEOMETRY & RADIAN MEASURE

IB Math SL1 - Santowski

Lesson Objectives

- (1) Change the way we understand angles ➔ from triangles to circles
- (2) Understand angles in co-ordinate geometry as angles in standard position and thereby introduce the "unit circle"
- (3) Introduce the radian as a new way to measure angles
- (4) Introduce a way of measuring the area of a sector, given our understanding of "radians"
- (5) Introduce a way to measure the length of an arc, given our understanding of "radians"

Fast Five

- Evaluate sin(50°) ➔ illustrate with a diagram
- Evaluate sin(130°) ➔ illustrate with a diagram
- Evaluate sin(230°) ➔ illustrate with a diagram??
- Evaluate sin(320°) ➔ illustrate with a diagram??
- Evaluate sin(770°) ➔ illustrate with a diagram??
- Evaluate sin(-50°) ➔ illustrate with a diagram??

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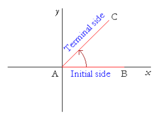
(A) Angles in Standard Position

- Angles in standard position are defined as angles drawn in the Cartesian plane where the initial arm of the angle is on the *x* axis, the vertex is on the origin and the terminal arm is somewhere in one of the four quadrants on the Cartesian plane
- To form angles of various measure, the terminal arm is simply rotated through a given angle
- http://www.analyzemath.com/Angle/Angle.html



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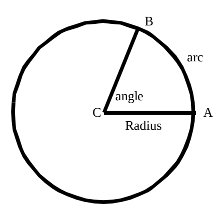
(B) Radians

- We can measure angles in several ways - one of which is degrees
- Another way to measure an angle is by means of radians
- One definition to start with ➔ an arc is a distance along the curve of the circle ➔ that is, part of the circumference
- One radian is defined as the measure of the angle subtended at the center of a circle by an arc equal in length to the radius of the circle
- Now, what does this mean?
- http://www.wgss.ca/ebalzarini/applets/angle_in_standard_position.html

(B) Radians

If we rotate a terminal arm (BC) around a given angle, then the end of the arm (at point B) moves along the circumference from A to B

If the distance point B moves is equal in measure to the radius, then the angle that the terminal arm has rotated is defined as one radian



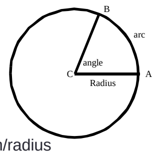
If B moves along the circumference a distance twice that of the radius, then the angle subtended by the arc is 2 radians

So we come up with a formula of θ = arc length/radius = s/r

(C) Converting between Degrees and Radians

If point B moves around the entire circle, it has revolved or rotated 360°

Likewise, how far has the tip of the terminal arm traveled? One circumference or $2\pi r$ units.



So in terms of radians, the formula is θ = arc length/radius
$\theta = s/r = 2\pi r/r = 2\pi$ radians

So then an angle of 360° = 2π radians or more easily, an angle of 180° = π radians

(C) Converting from Degrees to Radians

- Our standard set of first quadrant angles include 0°, 30°, 45°, 60°, 90° and we now convert them to radians:
- We can set up equivalent ratios as:
- 30° =
- 45° =
- 60° =
- 90° =

(C) Converting from Degrees to Radians

- Our standard set of first quadrant angles include 0°, 30°, 45°, 60°, 90° and we now convert them to radians:
- We can set up equivalent ratios as:
- 30°/ x radians = 180°/ π radians
- Then x = π /6 radians
- 45°/x = 180°/ π ➔ x = π /4 radians
- 60°/x = 180°/ π ➔ x = π /3 radians
- 90°/x = 180°/ π ➔ x = π /2 radians

(D) Converting from Radians to Degrees

- Let's work with our second quadrant angles with our equivalent ratios:
- $2\pi/3$ radians
- $3\pi/4$ radians
- $5\pi/6$ radians

(D) Converting from Radians to Degrees

- Let's work with our second quadrant angles with our equivalent ratios:
- 180°/ π = x / ($2\pi/3$)
  - ➔ x = ($2\pi/3$)(180/π) = 120°
- 180°/ π = x / ($3\pi/4$)
  - ➔ x = ($3\pi/4$)(180/π) = 135°
- 180°/ π = x / ($5\pi/6$)
  - ➔ x = ($5\pi/6$)(180/π) = 150°

(E) Table of Equivalent Angles

- We can compare the measures of important angles in both units on the following table:

0 °	90°	180°	270°	360°

(E) Table of Equivalent Angles

- We can compare the measures of important angles in both units on the following table:

0 °	90°	180°	270°	360°
0 rad	π/2 rad	π rad	3π/2 rad	2π rad

(E) Table of Equivalent Angles

- We can compare the measures of important angles in both units on the following table:

30	45	60	120	135	150	210	225	240	300	315	330

(E) Table of Equivalent Angles

- We can compare the measures of important angles in both units on the following table:

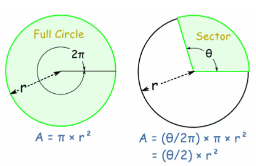
30	45	60	120	135	150	210	225	240	300	315	330
$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$	$7\pi/6$	$5\pi/4$	$4\pi/3$	$5\pi/3$	$7\pi/4$	$11\pi/6$

(F) Area of a Sector

- Recall the area of a circle formula: $A = \pi r^2$
- So how would you find the area of:
  - (i) half a circle
  - (ii) a quarter of a circle
  - (iii) one tenth of a circle
  - (iv) the area swept out by the terminal arm as it rotates 36 degrees?
  - (v) the area swept out by the terminal arm as it rotates 90 degrees?
  - (vi) the area swept out by the terminal arm as it rotates π /2 radians?

(F) Area of a Sector

- CONCLUSION from previous slide ➔ You can work out the Area of a Sector by comparing its angle to the angle of a full circle.
- Note: I am using radians for the angles.



(G) Arc Length

- Recall the circumference of a circle formula: $A = 2\pi r$
- So how would you find the arc length of:
  - (i) half a circle
  - (ii) a quarter of a circle
  - (iii) one tenth of a circle
  - (iv) the arc length traveled by the terminal arm as it rotates 36 degrees?
  - (v) the arc length traveled by the terminal arm as it rotates 90 degrees?
  - (vi) the arc length traveled by the terminal arm as it rotates π /2 radians?

(G) Arc Length

- Arc Length of Sector or Segment
- By the same reasoning, the arc length (of a Sector or Segment) is Arc Length "L" = **θ** × r
- = (**θ** × **π**/180) × r *(if **θ** is in degrees)*
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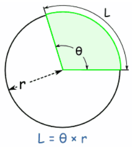

(H) Examples

(I) Internet Links

- Topics in trigonometry: Radian measure from The Math Page
- Measurement of angles from David Joyce, Clark University
- Radians and Degrees - on-line Math Lesson from TV

(J) Homework

- HW - Ex 13B.2 (all),
- Nelson text, p442, Q2,3,5,6