# Lesson 2 - Composition of Functions

Math SL1 - Santowski



## Fast Five

- Graphically determine the domain and range of g(x) = 1/x
- Graphically determine the domain and range of m(x) = 3x-5
- Graphically determine the domain and range of T(x) = 1/(3x-5)
- **Predict** the domain and range of d(x) = 1/(ax + b)



## Lesson Objectives

- (a) Understand **HOW** to compose functions
- (b) Understand how to compose functions when they are presented in **alternate representations**
- (c) Understand the **domains** of composite functions by considering the **relevant features of the constituent functions**



## The BIG Picture

- Sometimes working with multiple functions can be challenging, so **a single function** can be created by various algebraic combinations of the multiple functions
- Sometimes, complicated looking functions can be simplified by understanding that they are **made from** simpler, less complicated functions



## (A) Composition of Functions – An Example

- Andrew earns a daily wage of $20/h plus $15/d for travel expenses.
- (a) Write an equation for his daily earnings as Earnings =????.
- However, Andrew also pays union fees at 2.5% of his daily earnings.
- (b) Write an equation for his union fees as Fees = ???
- (c) Prepare a table of values showing the relationship between the hours worked ad the union fees paid

| Hours worked per day | Union Fees Paid |
|---|---|
| | |
| | |



## (A) Composition of Functions – An Example

- The following examples will be various ways of representing the composition of functions
- ex 1. Andrew earns a daily wage of $20/h plus $15/d for travel expenses.
- We can write his daily earnings as an equation as Earnings = 20h + 15.
- However, Andrew also pays union fees at 2.5% of his daily earnings, which we can write as the equation Fees = 0.025 x (daily earnings)
- We can demonstrate with a table of values

| Hours worked per day | Daily Earnings | Union Fees Paid |
|---|---|---|
| 2 | 20(2) + 15 = 55 | 0.025(55) = 1.375 |
| 5 | 20(5) + 15 = 115 | 0.025(115) = 2.875 |

## (A) Composition of Functions – An Example

- What we see is that the one function value (daily earnings or E) is being **substituted** into the second function (Fees = 0.025 x daily earnings) in order to generate the value for the union fees.
- We can generate a **direct formula** for the union fees by substituting the earnings function into the Fees function as follows: Fees = 0.025(20h + 15).
- Hence, the Fees function is called a composed function as Fees(daily earnings) = 0.025 x daily earnings



## (B) Review: Function Composition

- So we have a way of creating a new function ➔ we can **compose** two functions which is basically a **substitution of one function into another**.
- we have a notation that communicates this idea ➔ if ***f*(*x*)** is one functions and ***g*(*x*)** is a second function, then the composition notation is ➔ **f o g (x)**



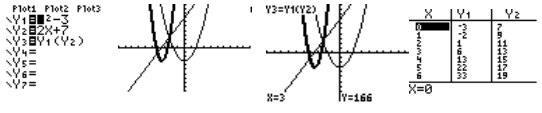
## (C) Composition of Functions – Example #1

- We can define *f* and *g* differently, this time as formulas:
- $f(x) = x^2 - 3$ and $g(x) = 2x + 7$
- We will try the following:
- (i) evaluate *f*(*g*(3)) or *fog*(3)
- (ii) evaluate *gof*(3) or *g*(*f*(3)) as well as *gof*(0)
- (iii) evaluate *fog*(x) and *gof*(x)
- (iv) evaluate *fog*(5) and *fog*(2)
- (v) evaluate *gof*(9) and *g*(*f*(7)) and *gog*(1)
- Repeat if f(x) = 1/x and g(x) = 3x - 6



## (C) Composition of Functions – Example #1

- How can I do the same thing on a TI-83/4??
- Again, let $f(x) = x^2 - 3$ and $g(x) = 2x + 7$
  - (i) evaluate *f*(*g*(3)) or *fog*(3)
  - (ii) evaluate *gof*(3) or *g*(*f*(3))






## (C) Composition of Functions – Example #2

- ex 2. We will now define *f* and *g* as follows:
- *f* = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}
- (i) evaluate *fog*(3)
- (ii) evaluate *fog* (1)
- (iii) evaluate *fog* (5) and see what happens ➔ why?
- (iv) evaluate *gof* (9) and *g*(*f*(7) and *gog* (1)

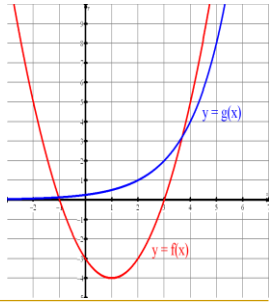


## (C) Composition of Functions – Example #2

- ex 2. We will now define *f* and *g* as follows:
- *f* = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}
- We will evaluate *fog*(3) (or f(g(3)) ➔ we will substitute a specific *g* function value into *f*, that of *g*(3).
- The *g* function value at *x* = 3 is 7 (i.e. g(3) = 7)
- So now we evaluate f at the new value of x = 7 ➔we look at our *f* data and see that *f*(7) = 4

## (C) Composition of Functions – Example #3

- We can define *f* and *g* differently, this time as graphs:
- We will try the following:
- (i) *f*(*g*(3)) or *fog*(3)
- (ii) *gof*(3) or *g*(*f*(3))
- (iii) evaluate *fog*(2) and *fog*(-1)
- (iv) evaluate *gof*(0) and *g*(*f*(1)) and *gog*(2)






## (D) DEcomposing Functions

- Our second BIG picture objective with function composition was to help us work with COMPLICATED looking functions and understand that they can be the result of function compositions
- Determine the functions that were composed to produce the following functions:
- Examples

(i) $y = 2^{x-4}$

(ii) $y = (2^x)^2 - 2 \cdot 2^x - 8$

(iii) $y = \dfrac{1}{x^2 - 4}$

(iv) $y = \sin(2(x-3))$

(v) $y = \cos\left(\dfrac{1}{(x-2)^2}\right)$



## (E) Applications

- Refrigeration slows down the growth of bacteria in food. The number of bacteria, B, is a function of the temperature, T, of the food. The function is $B(T) = 15T^2 - 70T + 600$. However, once the food is removed from the fridge, the temperature of the food, T, is a function of the time in hours since its been removed from the fridge, t according to the function $T(t) = 3.5 + 3$
  - (a) After 1.5 hours, how many bacteria are in the food?
  - (b) HOW will you determine when will the bacteria count reach 1200?
  - (c) Write an equation showing the relationship between the number of bacteria and the time since the food was removed from the fridge



## (F) Classwork

- Now go to the following link for our classwork:
- Composition of Functions Questions from AnalyzeMath



## (G) Internet Links

- Composition of Functions from Visual Calculus ➔ Read the Discussion
- Tutorial on Composition of Functions from AnalyzeMath
- Composition of Functions Lesson - From PurpleMath
- Operations with Functions from WTAMU ➔ includes examples with composition



## (H) Homework

- Textbook ➔ Fcn Composition ➔Ex 1D, pg27; Q2, 3,4
- Textbook ➔ Fcn Notation ➔ Ex 1C, pg24; Q5a,6,8,9
- Online Presumed Knowledge Quiz

## (D) Composition of Functions – New Domains

- When we create a new function from 2 or more functions (as in composing 2 functions), we must consider the domain of the new function
- So domain restrictions to remember from previous courses:
- Square root functions ➔ ???
- Reciprocal functions ➔ ?????
- Exponential, polynomial, absolute value ➔ ?????
- Now go back to the previous slides and state the domain of the new functions you created



## (D) Composition of Functions – New Domains

- When we create a new function from 2 or more functions (as in composing 2 functions), we must consider the domain of the new function
- So domain restrictions to remember from previous courses:
- Square root functions ➔ we take square roots of positive numbers
- Reciprocal functions ➔ denominators cannot be zero
- Exponential, polynomial, absolute value ➔ unrestricted domains
- Now go back to the previous slides and state the domain of the new functions you created