

Lesson 26 – Introduction to Sequences & Series

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Lesson Objectives

- ▶ The student will participate in an exploration into the nature & application of arithmetic and geometric sequences
- ▶ The student will learn the key terms associated with sequences
- ▶ The student will predict patterns in sequences and then write algebraic expressions for these patterns

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Exploration 1 – Simple Interest

- ▶ The formula for simple interest is $I = Prt$ and the formula to determine the amount of the investment is $A = P + I$. If we combine the 2 formulas, we get $A = P + Prt = P(1 + rt)$
- ▶ So you will explore the following investment scenario → you invest \$10,000 on which you earn 10% simple interest. You invest the money for 10 years.
- ▶ Complete the following table, in which you show the relationship between how much interest you earn and then the value of your total investment
- ▶ Prepare a scatter plot that shows the relationship between the number of years invested (time) and value of the investment.
- ▶ Determine an equation that summarizes the relationship between time of investment and value of the investment

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Exploration 2 – Compound Interest

- ▶ The formula for the total value of an investment earning compound interest is given below, where r is the annual interest rate, n is the number of times per year that interest is compounded, and t is the time of the investment.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

- ▶ So you will explore the following investment scenario → you invest \$10,000 on which you earn 10% interest, compounded annually for a period of 10 years.
- ▶ Complete the following table, in which you show the relationship between how much interest you earn and then the value of your total investment
- ▶ Prepare a scatter plot that shows the relationship between the number of years invested (time) and value of the investment.
- ▶ Determine an equation that summarizes the relationship between time of investment and value of the investment

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Exploration 3 – Earning on Commission

- ▶ So you will explore the following salary scenario → you are a salesperson and you sell motorcycles. Your salary is \$500 per month but you earn a commission of \$500 for every motorcycle you sell.
- ▶ Complete the following table, in which you show the relationship between how many motorcycles you sell and much salary you earn.
- ▶ Prepare a scatter plot that shows the relationship between the number of motorcycles sold and your salary.
- ▶ Determine an equation that summarizes the relationship between the number of motorcycles you sell and your salary.

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Exploration 4 – Rebound Heights

- ▶ So you will explore the following salary scenario → you release a ball from a height of 3 meters. As it bounces off the floor, it returns to a maximum rebound height of 60% of its previous maximum rebound height.
- ▶ Complete the following table, in which you show the relationship between how many bounces the ball has made and the maximum rebound height.
- ▶ Prepare a scatter plot that shows the relationship between how many bounces the ball has made and the maximum rebound height.
- ▶ Determine an equation that summarizes the relationship between how many bounces the ball has made and the maximum rebound height.

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Exploration 5 – Optimizing Revenues

- ▶ So you will explore the following salary scenario → Given a ticket price of \$25.00, you know you will sell 4000 seats to a charity performance. You also know that for every \$1.00 price increment, you sell 50 less seats.
- ▶ Complete the following table, in which you show the relationship between the number of price increments you make and the revenues made by the charity performance.
- ▶ Prepare a scatter plot that shows the relationship between the number of price increments you make and the revenues made by the charity performance.
- ▶ Determine an equation that summarizes the relationship between the number of price increments you make and the revenues made by the charity performance.

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(A) Definitions & Terms

- ▶ A **sequence** is a ordered set of numbers
 - ▶ **finite sequence**, meaning that it has a certain number of terms
 - ▶ **infinite sequence**, meaning it has an endless number of terms.
- ▶ Each number in the sequence is called a **term**.
- ▶ Each term is numbered and presented in the following notation: u_1 or t_1 is designated as the first term; u_n or t_n is referred to as the nth term.
- ▶ For example, in the sequence 3,5,8,2,5,89,4 → $u_4 = 2$ and is referred to as the 4th term.

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(A) Definitions & Terms

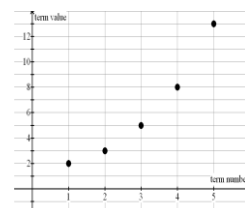
- ▶ The terms of a sequence may or may not have a specific pattern.
- ▶ If there is a pattern, then we can come up with a **rule or an algebraic expression** to describe every term of the sequence.
- ▶ One way to express this rule is called **the general term** of the sequence.

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(A) Definitions & Terms

- ▶ A sequence can be **graphically** visualized by plotting on a Cartesian plane, where the position of each term (the number of each term) is plotted on the x axis and the term value is plotted on the y axis.



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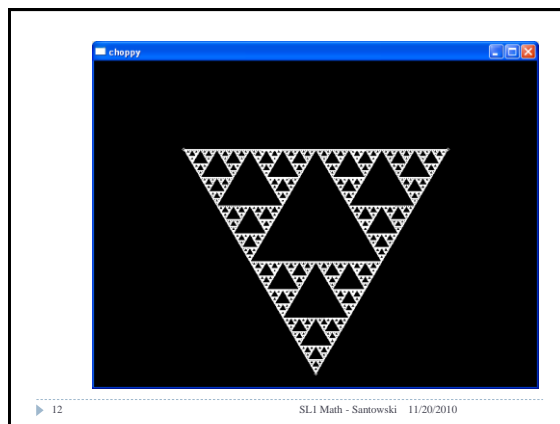
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(B) Examples of Sequences

- ▶ ex 5. Write a sequence that represents the total number of triangles formed in the following situation:
- ▶ To make a Sierpinski's triangle, you take an equilateral triangle, divide it into four congruent triangles by joining the midpoints of the sides of the larger triangle. Then repeat the process in each of the three newly created triangles to make a third pattern and so on.....

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(B) Examples of Sequences

- ▶ ex 1. Given the sequence 1,5,25,125,625,..... find the next four terms
- ▶ ex 2. Given a formula, find the first 5 terms
 - ▶ (i) $u_n = 3n - 2$
 - ▶ (ii) $u_n = n^2 - 1$
 - ▶ (iii) $u_n = (n-2)/(n+2)$
 - ▶ (iv) $u_n = 5^{n-1}$

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(B) Examples of Sequences

- ▶ ex 3. Find the formula for the n th term (or the general term), then list the next three terms:
 - ▶ (i) 5,10,15,20
 - ▶ (ii) 2,3,4,5,.....
 - ▶ (iii) 1,4,9,16,.....
 - ▶ (iv) 2,4,6,8,.....
 - ▶ (v) -3,-6,-12,-24,...
 - ▶ (vi) $1/2, 2/3, 3/4, 4/5, \dots$

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(C) Patterns in Sequences

- ▶ Comment upon any pattern you see in these sequences
- ▶ \$100, \$110, \$120, \$130, \$140, \$150,
- ▶ 8, 12, 16, 20, 24,.....
- ▶ 11, 9, 7, 5, 3,.....
- ▶ Comment upon any pattern you see in the sequences
- ▶ 2, 10, 50, 250,.....
- ▶ 5, -10, 20, -40, 80,.....
- ▶ 6, 0.6, 0.06, 0.006, 0.0006,.....
- ▶ 2, 4, 8, 16, 32, 64,
- ▶ 100, 50, 25, 12.5, 6.25, ...

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(C) Algebraic Formulas

- ▶ Given the sequence 8, 12, 16, 20, 24,, develop a formula that you can use to predict the 407th term
- ▶ EXPLAIN your prediction/formula
- ▶ Given the sequence 3, 6, 12, 24, 48,, develop a formula that you can use to predict the 41st term
- ▶ EXPLAIN your prediction/formula

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(E) Homework

- ▶ From the HH textbook:
 - ▶ Ex 2A #2ch, 5af;
 - ▶ Ex 2B #1f, 2c;
 - ▶ Ex 2C #1, 3, 5af, 6a, 7a, 8, 9

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