

Lesson 25 - Applications of Logarithmic Functions

Work through the following word problems, presenting complete algebraic & graphic solutions to the questions

1. Mr. S. drinks a cup of coffee at 9:45 am and his coffee contains 150 mg of caffeine. Since the half-life of caffeine for an average adult is 5.5 hours, determine how much caffeine is in Mr. S.'s body at class-time (1:10pm). Then determine how much time passes before I have 30 mg of caffeine in my body.
2. Two populations of bacteria are growing at different rates. Their populations at time t are given by $P_1(t) = P_0 5^{t+2}$ and $P_2(t) = P_0 e^{2t}$ respectively.
 - a. At what time are the populations the same?
 - b. When is the population of P_2 twice that of P_1 ?
3. Dry cleaners use a cleaning fluid that is purified by evaporation and condensation after each cleaning cycle. Every time it is purified, 2% of the fluid is lost
 - a. An equipment manufacturer claims that after 20 cycles, about two-thirds of the fluid remains. Verify or reject this claim.
 - b. If the fluid has to be "topped up" when half the original amount remains, after how many cycles should the fluid be topped up?
 - c. A manufacturer has developed a new process such that two-thirds of the cleaning fluid remains after 40 cycles. What percentage of fluid is lost after each cycle?
4. You invest \$5000 in a stock that grows at a rate of 12% per annum compounded quarterly. The value of the stock is given by the equation $V = 5000(1 + 0.12/4)^{4x}$, or $V = 5000(1.03)^{4x}$ where x is measured in years.
 - a. Find the value of the stock in 6 years.
 - b. Find when the stock value is \$14,000
5. The population of a small town was 35,000 in 1980 and in 1990, it was 57,010. Create an algebraic model for the town's population growth. Check your model using the fact that the population was 72800 in 1995. What will the population be in 2010? When will the population be 100,000?
6. The sales S (in thousands of units) of a new CD burner after it has been on the market for t years are given by $S(t) = 100(1 - e^{-kt})$. Fifteen thousand units of the new product were sold the first year.
 - a. Complete the model by solving for k .
 - b. Use your calculator to graph the model.
 - c. Determine $S(3)$ and interpret.
 - d. When were 8,000 units sold?

7. A conservation authority releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1,000 animals and that the growth of the herd will follow the logistic curve $p(t) = \frac{1000}{1 + 9e^{-0.165t}}$, where t is time measured in months.
- Graph the function and determine the values of p at which the horizontal asymptotes occur. Interpret the meaning of the asymptotes in the context of the problem.
 - Estimate the population after 5 years.
 - When will the population reach 500?
8. A coroner is called in to investigate a death. The coroner determines the body's temperature at 9:00 to be 85.7°F and at 11:00 the temperature was 82.8°F. The relationship between time elapsed since death (t , in hours) and the body temperature (T in degrees Fahrenheit) is $t(T) = -10 \ln \frac{T - 70}{98.6 - 70}$. (assume that the body had a normal body temperature of 98.6°F and the room temperature was 70°F). Estimate the time of death of the body.
9. The model $L(p) = 12.542 \ln \left(\frac{p}{p - 1000} \right)$, $p > 1000$ approximates the length of a home mortgage of \$150,000 at 8% in terms of the monthly payment. In the model, L is the length of the mortgage in years and p is the monthly payments in dollars.
- Use the model to approximate the length of the mortgage when the monthly payments are \$1254.68
 - Approximate the total amount paid over the term of the mortgage from (a). What amount of the total is interest costs?
 - If I want to pay off the mortgage in 17 years, how much should my monthly payments be?
10. In a psychology project on learning theory, a mathematical model for the proportion of correct responses, P , after n trials was found to be $P(n) = \frac{0.83}{1 + e^{-0.2n}}$.
- Graph the function
 - Determine the equation(s) of any horizontal asymptotes and interpret in the context of the problem.
 - Determine $P(0)$ and $P(12)$ and interpret.
 - After how many trials will 60% of the responses be correct?
11. The demand function for a camera is given by $p(x) = 500 - 0.5(e^{0.004x})$, where p is the price of the camera in dollars and x is the demand (how many units can be sold at that price).
- Determine $p(1600)$ and interpret in the context of the problem.
 - Determine the demand, x , for a price of (i) \$600 and (ii) \$400