

Lesson 22 - Laws of Logarithms

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Lesson Objectives

- Understand the rationale behind the "laws of logs"
- Apply the various laws of logarithms in solving equations and simplifying expressions

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Summary of Laws

Logs as exponents	$b^{\log_b x} = x$
Product Rule	$\log_a(mn) = \log_a m + \log_a n$
Quotient Rule	$\log_a(m/n) = \log_a(m) - \log_a(n)$
Power Rule	$\log_a(m^p) = (p) \times (\log_a m)$

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(A) Properties of Logarithms – Product Law

- Recall the laws for exponents → product of powers → $(b^x)(b^y) = b^{(x+y)}$ → so we ADD the exponents when we multiply powers
- For example → $(2^3)(2^5) = 2^{(3+5)}$
- So we have our POWERS → $8 \times 32 = 256$

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(A) Properties of Logarithms – Product Law

- Now, let's consider this from the INVERSE viewpoint
- We have the ADDITION of the exponents
- $3 + 5 = 8$
- But recall from our work with logarithms, that the exponents are the OUTPUT of logarithmic functions
- So → $3 + 5 = 8$ becomes $\log_2 8 + \log_2 32 = \log_2 256$
- Now, HOW do we get the right side of our equation to equal the left?
- Recall that $8 \times 32 = 256$
- So $\log_2(8 \times 32) = \log_2 8 + \log_2 32 = \log_2 256$

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(A) Properties of Logarithms – Product Law

- So we have our first law → when adding two logarithms, we can simply write this as a single logarithm of the product of the 2 powers
- $\log_a(mn) = \log_a m + \log_a n$
- $\log_a m + \log_a n = \log_a(mn)$

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(A) Properties of Logarithms – Formal Proof of Product Law

- Express $\log_a m + \log_a n$ as a single logarithm
- We will let $\log_a m = x$ and $\log_a n = y$
- So $\log_a m + \log_a n$ becomes $x + y$
- But if $\log_a m = x$, then $a^x = m$ and likewise $a^y = n$
- Now take the product $(m)(n) = (a^x)(a^y) = a^{x+y}$
- Rewrite $mn = a^{x+y}$ in log form $\rightarrow \log_a(mn) = x + y$
- But $x + y = \log_a m + \log_a n$
- So thus $\log_a(mn) = \log_a m + \log_a n$

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(B) Properties of Logarithms – Quotient Law

- Recall the laws for exponents \rightarrow Quotient of powers $\rightarrow (b^x)/(b^y) = b^{(x-y)} \rightarrow$ so we subtract the exponents when we multiply powers
- For example $\rightarrow (2^8)/(2^3) = 2^{(8-3)}$
- So we have our POWERS $\rightarrow 256 \div 8 = 32$

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(B) Properties of Logarithms – Quotient Law

- Now, let's consider this from the INVERSE viewpoint
- We have the SUBTRACTION of the exponents
- $8 - 3 = 5$
- But recall from our work with logarithms, that the exponents are the OUTPUT of logarithmic functions
- So $\rightarrow 8 - 3 = 5$ becomes $\log_2 256 - \log_2 8 = \log_2 32$
- Now, HOW do we get the right side of our equation to equal the left?
- Recall that $256/8 = 32$
- So $\log_2(256/8) = \log_2 256 - \log_2 8 = \log_2 32$

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(B) Properties of Logarithms – Quotient Law

- So we have our second law \rightarrow when subtracting two logarithms, we can simply write this as a single logarithm of the quotient of the 2 powers
- $\log_a(m/n) = \log_a m - \log_a n$
- $\log_a m - \log_a n = \log_a(m/n)$

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(C) Properties of Logarithms- Logarithms of Powers

- Now work with $\log_3(625) = \log_3(5^4) = x :$
- we can rewrite as $\log_3(5 \times 5 \times 5 \times 5) = x$
- we can rewrite as $\log_3(5) + \log_3(5) + \log_3(5) + \log_3(5) = x$
- We can rewrite as $4 [\log_3(5)] = 4 \times 1 = 4$
- So we can generalize as $\log_3(5^4) = 4 [\log_3(5)]$
- So if $\log_3(625) = \log_3(5^4) = 4 \times \log_3(5) \rightarrow$ It would suggest a rule of logarithms $\rightarrow \log_a(b^x) = x \log_a b$

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(D) Properties of Logarithms – Logs as Exponents

- Consider the example $3^{\log_3 5} = x$
- Recall that the expression $\log_3(5)$ simply means "the exponent on 3 that gives 5" \rightarrow let's call that y
- So we are then asking you to place that same exponent (the y) on the same base of 3
- Therefore taking the exponent that gave us 5 on the base of 3 (y) onto a 3 again, must give us the same 5!!!!
- We can demonstrate this algebraically as well

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(D) Properties of Logarithms – Logs as Exponents

- Let's take our exponential equation and write it in logarithmic form
- So $3^{\log_3 5} = x$ becomes $\log_3(x) = \log_3(5)$
- Since both sides of our equation have a \log_3 then $x = 5$ as we had tried to reason out in the previous slide
- So we can generalize that $b^{\log_b x} = x$

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(E) Summary of Laws

Logs as exponents	$b^{\log_b x} = x$
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(F) Examples

- (i) $\log_3 54 + \log_3(3/2)$
- (ii) $\log_2 144 - \log_2 9$
- (iii) $\log 30 + \log(10/3)$
- (iv) which has a greater value
 - (a) $\log_3 72 - \log_3 8$ or (b) $\log 500 + \log 2$
- (v) express as a single value
 - (a) $3\log_2 x + 2\log_2 y - 4\log_2 a$
 - (b) $\log_3(x+y) + \log_3(x-y) - (\log_3 x + \log_3 y)$
- (vi) $\log_2(3/4) - \log_2(24)$
- (vii) $(\log_2 5 + \log_2 25.6) - (\log_2 16 + \log_2 9)$

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(F) Examples

- Solve for x
- Solve for x and verify your solution

$$\log_2 x = 2\log_2 7 + \log_2 3$$

$$\log_2 x + \log_2 11 = \log_2 \sqrt{99}$$

$$\log \sqrt[3]{x} + \log 13 = -\log \frac{1}{91}$$

$$\log_5(x+1) + \log_5 3 = 2$$

$$\log_3(x-2) + \log_3 x = 1$$

$$\log x + \log(x-5) = \log(2x-12)$$

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(F) Examples

- Solve and verify
- If $a^2 + b^2 = 23ab$, prove that

$$\log_2 x = \frac{1}{3}\log_2 3 + \log_2 \sqrt{3}$$

$$\log_5(x-1) - \log_5(x-5) = \log_5 \frac{1}{x+3}$$

$$\log 250 - \log 2 = 3\log \frac{1}{x}$$

$$\log\left(\frac{a+b}{5}\right) = \frac{\log a + \log b}{2}$$

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(F) Examples

- Write each single log expression as a sum/difference/product of logs

$$\log \frac{abc^2}{d^3}$$

$$\log \frac{x\sqrt[3]{y}}{z^5}$$

$$\ln \sqrt{\sin x \ln x}$$

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(F) Examples

Use the logarithm laws to simplify the following:

- $\log_2 xy - \log_2 x^2$
- $\log_2 \frac{8x^2}{y} + \log_2 2xy$
- $\log_3 9xy^2 - \log_3 27xy$
- $\log_4 (xy)^3 - \log_4 xy$
- $\log_3 9x^4 - \log_3 (3x)^2$

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(F) Examples

Exercise 2. Given $\text{Log}_{10}(5.0) = 0.70$ $\text{Log}_{10}(2.0) = 0.30$ $\text{Log}_{10}(3.0) = 0.48$, without a calculator, determine:

- | | |
|---|--|
| (1) $\text{Log}_{10}(6.0)$ | (6) $\text{Log}_{10}(0.40)$ |
| (2) $\text{Log}_{10}(8.0)$ | (7) $\text{Log}_{10}\left(\frac{1}{16}\right)$ |
| (3) $\text{Log}_{10}\left(\frac{1}{2}\right)$ | (8) $\text{Log}_{10}(\sqrt{5.0})$ |
| (4) $\text{Log}_{10}(15.)$ | (9) $\text{Log}_{10}(\sqrt[3]{3.0})$ |
| (5) $\text{Log}_{10}\left(\frac{1}{3}\right)$ | (10) $\text{Log}_{10}(0.036)$ |

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(G) Internet Links

- [Logarithm Rules Lesson from Purple Math](#)
- [College Algebra Tutorial on Logarithmic Properties from West Texas AM](#)

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(H) Homework

- Nelson textbook, p125, Q1-14
- HW
- Ex 4C#1acgklo, 2adi, 3, 4ade;#7acde, 8, 9, 10
- Ex 5C #1dgikl,2bdfh, 3ceg
- IB Packet #2, 3, 7, 8ab, 9a

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