

Lesson 19 - Solving Exponential Equations

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Lesson Objectives

- (1) Establish a context for the solutions to exponential equations
- (2) Review & apply strategies for solving exponential eqns:
 - (a) guess and check
 - (b) graphic
 - (c) algebraic
 - (i) rearrange eqn into equivalent bases
 - (ii) isolate parent function and apply inverse

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Lesson Objective #1 – Context for Exponential Equations

- (1) Establish a context for the solutions to exponential equations
- (a) population growth
- (b) decay

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(A) Context for Equations

- Write and then solve equations that model the following scenarios:
- ex. 1 A bacterial strain doubles every 30 minutes. How much time is required for the bacteria to grow from an initial 100 to 25,600?
- ex 2. The number of bacteria in a culture doubles every 2 hours. The population after 5 hours is 32,000. How many bacteria were there initially?

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(A) Context for Equations

- Write and then solve equations that model the following scenarios:
- Ex 1. 320 mg of iodine-131 is stored in a lab for 40d. At the end of this period, only 10 mg remains.
 - (a) What is the half-life of I-131?
 - (b) How much I-131 remains after 145 d?
 - (c) When will the I-131 remaining be 0.125 mg?
- Ex 2. Health officials found traces of Radium F beneath P044. After 69 d, they noticed that a certain amount of the substance had decayed to $1/\sqrt{2}$ of its original mass. Determine the half-life of Radium F

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Lesson Objective #2 – Solving Eqns

- Review & apply strategies for solving exponential eqns:
 - (a) guess and check
 - (b) graphic
 - (c) algebraic
 - (i) rearrange eqn into equivalent bases
 - (ii) isolate parent function and apply inverse

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(A) Solving Strategy #1 – Guess and Check

- Solve $5^x = 53$ using a guess and check strategy
- Solve $2^x = 3$ using a guess and check strategy

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(A) Solving Strategy #1 – Guess and Check

- Solve $5^x = 53$ using a guess and check strategy
- we can simply “guess & check” to find the right exponent on 5 that gives us 53 → we know that $5^2 = 25$ and $5^3 = 125$, so the solution should be somewhere closer to 2 than 3
- Solve $2^x = 3$ using a guess and check strategy
- we can simply “guess & check” to find the right exponent on 2 that gives us 3 → we know that $2^1 = 2$ and $2^2 = 4$, so the solution should be somewhere between to 1 and 2

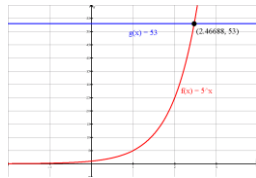
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(B) Solving Strategy #2 – Graphic Solutions

- Going back the example of $5^x = 53$, we always have the graphing option
- We simply graph $y_1 = 5^x$ and simultaneously graph $y_2 = 53$ and look for an intersection point (2.46688, 53)



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(B) Solving Strategy #2 – Graphic Solutions

- Solve the following equations graphically.
 - (i) $8^x = 2^{x+1}$
 - (ii) $5^x = 53$
 - (iii) $2^x = 3$
 - (iv) $2^{4x-1} = 3^{1-x}$

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(C) Solving Strategy #3 – Algebraic Solutions

- We will work with 2 strategies for solving exponential equations:
 - (a) Rearrange the equations using various valid algebraic manipulations to either (i) make the bases equivalent or (ii) make the exponents equivalent
 - (b) Isolate the parent exponential function and apply the inverse function to “unexponentiate” the parent function

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(D) Algebraic Solving Strategies – Math Principles

- MATH PRINCIPLE → PROPERTIES OF EQUALITIES
 - If two powers are equal and they have the same base, then the exponents must be the same
 - ex. if $b^x = a^y$ and $a = b$, then $x = y$.
 - If two powers are equal and they have the same exponents, then the bases must be the same
 - ex. if $b^x = a^y$ and $x = y$, then $a = b$.

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(E) Solving Strategies – Algebraic Solution #1

- This prior observation set up our general equation solving strategy => get both sides of an equation expressed in the same base
- ex. Solve and verify the following:
 - (a) $(\frac{1}{2})^x = 4^{2-x}$ (b) $3^{y+2} = 1/27$
 - (c) $(1/16)^{2a-3} = (1/4)^{a+3}$ (d) $3^{2x} = 81$
 - (e) $5^{2x-1} = 1/125$ (f) $36^{2x+4} = \sqrt{(1296^x)}$

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(E) Solving Strategies – Algebraic Solution #1

- The next couple of examples relate to composed functions → quadratic fcn's composed with exponential fcn's:
- Ex: Let $f(x) = 2^x$ and let $g(x) = x^2 + 2x$, so solve $fog(x) = \frac{1}{2}$
- Ex: Let $f(x) = x^2 - x$ and let $g(x) = 2^x$, so solve $fog(x) = 12$

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(E) Solving Strategies – Algebraic Solution #1

- The next couple of examples relate to composed functions → quadratic fcn's composed with exponential fcn's:
- Ex: Let $f(x) = 2^x$ and let $g(x) = x^2 + 2x$, so solve $fog(x) = \frac{1}{2}$ → i.e. Solve $2^{x^2+2x} = \frac{1}{2}$
- Ex: Let $f(x) = x^2 - x$ and let $g(x) = 2^x$, so solve $fog(x) = 12$ → i.e. Solve $2^{2x} - 2^x = 12$

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(E) Solving Strategies – Assorted Examples

- Solve the following for x using the most appropriate method:
 - (a) $2^x = 8$ (b) $2^x = 1.6$
 - (c) $2^x = 11$ (d) $2^x = 12$
 - (e) $2^{4x+1} = 8^{1-x}$ (f) $2^{x^2-4} = 8^x$
 - (g) $2^{3x+2} = 9$ (h) $3(2^{2x-1}) = 4^{-x}$
 - (i) $2^{4y+1} - 3^y = 0$

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(F) Examples with Applications

- Example 1 → Radioactive materials decay according to the formula $N(t) = N_0(1/2)^{t/h}$ where N_0 is the initial amount, t is the time, and h is the half-life of the chemical, and the $(1/2)$ represents the decay factor. If Radon has a half life of 25 days, how long does it take a 200 mg sample to decay to 12.5 mg?

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(F) Examples with Applications

- Example 2 → A bacterial culture doubles in size every 25 minutes. If a population starts with 100 bacteria, then how long will it take the population to reach 2,000,000?

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(F) Examples with Applications

- Two populations of bacteria are growing at different rates. Their populations at time t are given by $P_1(t) = 5^{t+2}$ and $P_2(t) = e^{2t}$ respectively. At what time are the populations the same?

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(F) Examples with Applications

- ex 1. Mr. S. drinks a cup of coffee at 9:45 am and his coffee contains 150 mg of caffeine. Since the half-life of caffeine for an average adult is 5.5 hours, determine how much caffeine is in Mr. S.'s body at class-time (1:10pm). Then determine how much time passes before I have 30 mg of caffeine in my body.
- ex 2. The value of the Canadian dollar, at a time of inflation, decreases by 10% each year. What is the half-life of the Canadian dollar?

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(F) Examples with Applications

- ex 3. The half-life of radium-226 is 1620 a. Starting with a sample of 120 mg, after how many years is only 40 mg left?
- ex 4. Find the length of time required for an investment of \$1000 to grow to \$4,500 at a rate of 9% p.a. compounded quarterly.

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(F) Examples with Applications

- ex 5. Dry cleaners use a cleaning fluid that is purified by evaporation and condensation after each cleaning cycle. Every time it is purified, 2% of the fluid is lost
 - (a) An equipment manufacturer claims that after 20 cycles, about two-thirds of the fluid remains. Verify or reject this claim.
 - (b) If the fluid has to be "topped up" when half the original amount remains, after how many cycles should the fluid be topped up?
 - (c) A manufacturer has developed a new process such that two-thirds of the cleaning fluid remains after 40 cycles. What percentage of fluid is lost after each cycle?

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(H) Homework

- H&M, Math 12, S2.3, p57-8, Q1,3,4,6,8-10ab each
- IB Textbook:
 - Ex 4D #1fg, 2bf;
 - Ex 4E #4;
 - Ex 5D #1bh;
 - Ex 5E #1a,3c

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