Lesson 18 – Algebra of Exponential Functions – Rational Exponent

Math SL1 - Santowski

(A) Review of Modeling Example #3

- The following data table shows the historic world population since 1950:

Year	1950	1960	1970	1980	1990	1995	2000	2005	2010
Pop (in millions)	2.56	3.04	3.71	4.45	5.29	5.780	6.09	6.47	6.85

- Mr S proposes the following variation, given his definition of the variables:

# of decades since 1950	0	1	2	3	4	5	6
Pop (in millions)	2.56	3.04	3.71	4.45	5.29	6.09	6.85

(A) Review of Modeling Example #3

- Mr S proposes the following equation development:

# of decades since 1950	0	1	2	3	4	5	6
Pop (in millions)	2.56	3.04	3.71	4.45	5.29	6.09	6.85

- So my equation is P(t) = 2.56(1.179)t
- Interpret the meaning of the base of 1.18
- Explain algebraically how to determine the annual growth rate.
- Predict the population in 1953.

(A) Review of Modeling Example #3

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- So my equation is P(t) = 2.56(1.179)t
- To predict the population in 1953 ➔ explain the MEANING of the exponent 3/10 in the equation $y = 2.56(1.179)^{(3/10)}$

Terminology (Santowski's Take)

- In the expression $2^3 = 8$ ➔
- a) the BASE is 2: the base is the number that is repeatedly multiplied by itself.
- b) the EXPONENT is 3: the exponent is the number of times that the base is multiplied by itself.
- c) the POWER is 8: the power is the ANSWER of the base raised to an exponent, or the product of repeatedly multiplying the base by itself an exponent number of times.

(A) Review of Exponent Laws

- product of powers: $3^4 \times 3^6$
- $3^4 \times 3^6 = 3^{4+6}$ ➔ add exponents if bases are **equal**
- quotient of powers: $3^9 \div 3^2$
- $3^9 \div 3^2 = 3^{9-2}$ ➔ subtract exponents if bases are **equal**
- power of a power: $(3^2)^4$
- $(3^2)^4 = 3^{2 \times 4}$ ➔ multiply powers
- power of a product: $(3 \times a)^5$
- $(3 \times a)^5 = 3^5 \times a^5 = 243a^5$ ➔ distribute the exponent
- power of a quotient: $(a/3)^5$
- $(a/3)^5 = a^5 \div 3^5 = a^5/243$ ➔ distribute the exponent

(B) Review of Zero & Negative Exponent

- PROVE that $2^0 = 1$.
- And then
- Prove that, in general then $b^0 = 1$
- Prove that $2^{-4} = 1/16$
- And then
- Prove that, in general then $b^{-e} = 1/b^e$

(C) Review of Rational Exponent

- Use the Law of Exponents to show that $9^{½} = \sqrt{9}$.

(C) Review of Rational Exponent

- We will use the Law of Exponents to prove that $9^{½} = \sqrt{9}$.
- $9^{½} \times 9^{½} = 9^{(½ + ½)} = 9^1$
- Therefore, $9^{½}$ is the positive number which when multiplied by itself gives 9 ➔
- The only number with this property is 3, or $\sqrt{9}$ or $\sqrt[2]{9}$
- So what does it mean? It means we are finding the second root of 9 ➔ $\sqrt[2]{9}$

(C) Review of Rational Exponent

- We can go through the same process to develop a meaning to $27^{1/3}$
- $27^{1/3} \times 27^{1/3} \times 27^{1/3} = 27^{(1/3 + 1/3 + 1/3)} = 27^1$
- Therefore, $27^{1/3}$ is the positive number which when multiplied by itself three times gives 27
- The only number with this property is 3, or $\sqrt[3]{27}$ or the third root of 27
- In general $b^{\frac{1}{n}} = \sqrt[n]{b}$ which means we are finding the nth root of b.

(D) The Rational Exponent m/n

- We can use our knowledge of Laws of Exponents to help us solve $b^{m/n}$
- ex. Rewrite $32^{3/5}$ making use of the Power of powers >>> $(32^{1/5})^3$
- so it means we are looking for the 5th root of 32 which is 2 and then we cube it which is 8
- In general, $b^{\frac{m}{n}} = \sqrt[n]{(b^m)}$ or $b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{b}\right)^m$

(E) Examples

- We will use the various laws of exponents to simplify expressions.
- ex. $27^{1/3}$
- ex. $(-32)^{0.4}$
- ex. $81^{-3/4}$
- ex. Evaluate $49^{1.5} + 64^{-1/3} - 27^{-2/3}$
- ex. Evaluate $4^{1/2} + (-8)^{-1/3} - 27^{4/3}$
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- ex. Evaluate $\sqrt[3]{8} + \sqrt[4]{16} - 125^{-4/3}$
- ex. Evaluate $(4/9)^{½} + (4/25)^{3/2}$

(G) Applications

- Ex 1. The value of an investment, A, after *t* years is given by the formula $A(t) = 1280(1.085)^t$
  - (a) Determine the value of the investment in 6 ½ and in 12 ¼ years
  - (b) How many years will it take the investment to triple in value?

(G) Internet Links

- From West Texas A&M - Integral Exponents
- From West Texas A&M - Rational Exponents

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(H) Homework

- HW

- Ex 3A #1;
- Ex 3B #1efhi;
- Ex 3C #1fh, 2dg, 3cg, 4hip,
- 6dh, 7g, 8fh, 9dj, 10cjmnl, 11hklp, 12fip, 13
- Ex 3D #1ag, 2d, 3ceg,4d, 5c, 6agj;
- Ex 3E #1aef, 2ajk
- Ex 3F #1hijkl, 2dghijlm, 3bc

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