

Lesson 17 – Algebra of Exponential Functions – Exponent Laws

Math SL1 - Santowski

Transformations of Exponential Functions

- Mr Nourse and Mr S are having a debate about transformed exponential functions. Mr N says that the function $y = 4(2)^x$ represents a vertical stretch by a factor of 4 and I say that it represents a horizontal translation of 2 units to the left!!!
- Algebraically** justify either Mr N's opinion or Mr S's opinion

Applications → “Cooling” Functions

- In our Transformations mini-portfolio, the equations proposed for the cooling of coffee data was

$$T(t) = 0.21 \cdot 2^{\left(\frac{-1}{40}(t-317)\right)} + 26.9$$

- However, Mr S does an algebraic “Ratio” analysis and decides that the equation should be

$$T(t) = 51(0.98)^x + 27$$

- ALGEBRAICALLY Show that the 2 equations are essentially IDENTICAL!!

The Logistic Model

- In your portfolio, you are modeling with the logistic function as

$$y = \frac{K}{1 + Le^{-Mx}}$$

- Algebraically show WHY the parameter L is a HORIZONTAL TRANSLATION

The Logistic Model

- In your portfolio, you are modeling with the logistic function as

$$y = \frac{K}{1 + Le^{-Mx}}$$

- However, I gave you a link to AnalyzeMath.com that shows the logistic equation in transformational form as

$$y = \frac{a}{1 + be^{\frac{(x-c)}{d}}}$$

- Algebraically, show that the 2 equations are identical!!!

Terminology (Santowski's Take)

- In the expression $2^3 = 8 \rightarrow$

- a) the BASE is 2: the base is the number that is repeatedly multiplied by itself.
- b) the EXPONENT is 3: the exponent is the number of times that the base is multiplied by itself.
- c) the POWER is 8: the power is the ANSWER of the base raised to an exponent, or the product of repeatedly multiplying the base by itself an exponent number of times.

(A) Review of Exponent Laws

- product of powers: $3^4 \times 3^6$
- $3^4 \times 3^6 = 3^{4+6} \rightarrow$ add exponents if bases are **equal**
- quotient of powers: $3^9 \div 3^2$
- $3^9 \div 3^2 = 3^{9-2} \rightarrow$ subtract exponents if bases are **equal**
- power of a power: $(3^2)^4$
- $(3^2)^4 = 3^{2 \times 4} \rightarrow$ multiply powers
- power of a product: $(3 \times a)^5$
- $(3 \times a)^5 = 3^5 \times a^5 = 243a^5 \rightarrow$ distribute the exponent
- power of a quotient: $(a/3)^5$
- $(a/3)^5 = a^5 \div 3^5 = a^5/243 \rightarrow$ distribute the exponent

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(B) Review of Zero & Negative Exponent

- PROVE that $2^0 = 1$.
- And then
- Prove that, in general then $b^0 = 1$
- Prove that $2^{-4} = 1/16$
- And then
- Prove that, in general then $b^{-e} = 1/b^e$

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(B) Review of Zero & Negative Exponent

- Evaluate $2^5 \div 2^5$.
- (i) $2^5 \div 2^5 = 2^{5-5} = 2^0$
OR
- (ii) $2^5 \div 2^5 = 32 \div 32 = 1$
- Conclusion $\rightarrow 2^0 = 1$.
- In general then $b^0 = 1$
- Evaluate $2^3 \div 2^7$.
- (i) $2^3 \div 2^7 = 2^{3-7} = 2^{-4}$
- (ii) $2^3 \div 2^7 = 8 \div 128 = 1/16 = 1/2^4$
- Thus $\rightarrow 2^{-4} = 1/16 = 1/2^4$
- In general then $b^{-e} = 1/b^e$

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(F) Examples

- ex 1. Simplify the following expressions:
 - (i) $(3a^2b)(-2a^3b^2)$
 - (ii) $(2m^3)^4$
 - (iii) $(-4p^3q^2)^3$
- ex 2. Simplify $(6x^5y^3/8y^4)^2$
- ex 3. Simplify $(-6x^2y)(-9x^5y^2) / (3x^2y^4)$ and express answer with positive exponents
- ex 4. Evaluate the following
 - (i) $(3/4)^{-2}$
 - (ii) $(-6)^0 / (2^{-3})$
 - (iii) $(2^{-4} + 2^{-6}) / (2^{-3})$

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Examples

- <http://www.edhelper.com/exponents5.htm>
- <http://www.edhelper.com/exponents6.htm>
- <http://www.edhelper.com/exponents7.htm>
- <http://www.mathsisfun.com/algebra/exponent-laws.html>

(F) Examples

- Expand and simplify the following:

$$(i) x^{-\frac{1}{2}} \left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \right)$$

$$(ii) (2^x + 3)(2^{x+1} + 1)$$

$$(iii) (3^x - 3^{-x})^2$$

(O) Applications

- Ex 1. The value of an investment, A , after t years is given by the formula $A(t) = 1280(1.085)^t$
 - (a) Determine the value of the investment in 5 and in 10 years
 - (b) How many years will it take the investment to triple in value?

(G) Internet Links

- [From West Texas A&M - Integral Exponents](#)
- [From West Texas A&M - Rational Exponents](#)

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(H) Homework

- HW
 - Ex 3A #1;
 - Ex 3B #1efhi;
 - Ex 3C #1fh, 2dg, 3cg, 4hip,
 - 6dh, 7g, 8fh, 9dj, 10cjml, 11hkpl, 12fip, 13
 - Ex 3D #1ag, 2d, 3ceg, 4d, 5c, 6agj;
 - Ex 3E #1aef, 2ajk
 - Ex 3F #1hijkl, 2dghijlm, 3bc

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