Lesson 14 - The Inverse Function

Math SL1 - Santowski

Fast Five – Skills Preview

- Define "inverse of a function"
- Graphically, explain what "inverse of a function" means.
- Find the inverse of:
- (a) $3x - 2y + 7 = 0$
- (b) $y = \frac{1}{2}(x + 2)^2 - 5$
- (c) $h(x) = \frac{2}{3-x}$

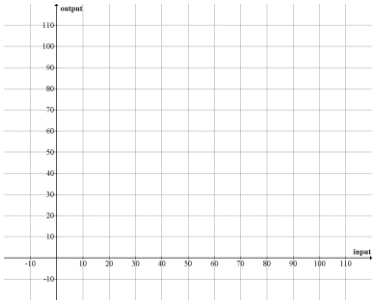
Lesson Objectives

- Find the inverse of a function from numeric/tabular, graphic or algebraic data
- Determine whether the inverse is or is not a function
- Understand inverses as transformations
- Compose a function with its inverse to develop the identity function

Opening Exercise - CONTEXT

- Consider is the idea of a relationship between °F and °C
- On a grid, graph the points (-10,10), (0,30), (10,50), (20,70), (30,90), (40,110)
- Then prepare a mapping diagram using the same points
- Can you determine a mathematical relationship between x & y??

Opening Exercise



(A) Inverses - The Concept

- Let's back to our input ➔ output notion for functions.
- If functions are nothing more than input/output operators, then the concept of an inverse has us considering how to go in reverse ➔ how do you get from the output back to the input?

(B) Inverses - An Example of the Concept

- Consider is the idea of a relationship between °F and °C ➔ the relationship says that if we know °C we can convert to °F by means of the simplified formula ➔ double the temperature in °C and add 30°.
- If we work with this simplified formula ➔ Order pairs would include (-10,10), (0,30), (10,50), (20,70), (30,90)
- in our input/output formula idea we would have: input ➔ times 2 ➔ add 30 ➔ output

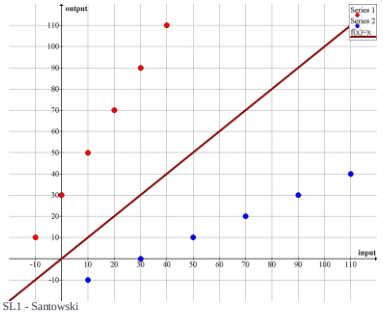
- if we wish to discuss the idea of an inverse, we would ask "how do we go from °F back to °C?"
- So to express the inverse, we would switch or reverse the ordered pairs or our input/output notion ➔ ordered pairs are(10,-10), (30,0), (50, 10), (70, 20), (90, 30) etc.…
- Our input/output formula idea would have us: do WHAT??

Opening Exercise - Revisited

- On a grid, graph the points (-10,10), (0,30), (10,50), (20,70), (30,90), (40,110)
- Determine a mathematical relationship between x & y
- Prepare a mapping diagram using the same points
- Graph the line y = x on the same graph

- Now, list the ordered pairs that are part of the "reverse" process or rather the inverse ➔ ……..
- Then graph these points of the inverse relation
- Then determine the mathematical relationship between x & y

Opening Exercise - Revisited



(C) Definition of an Inverse

- If the elements of the ordered pairs or mappings of a function are **reversed**, the resulting set of ordered pairs or mappings are referred to as the INVERSE.

- Since we are **REVERSING** the elements ➔another point worth noting: the domain of the original function now becomes the range of the inverse; likewise, the range of the original becomes the domain of the inverse.

(D) Notation of the Inverses

- The notation used is $f^{-1}(x)$ and it is used when the inverse relation IS in fact a function

- IMPORTANT NOTE: $f^{-1}(x)$ does not mean $(f(x))^{-1}$ or $1 / f(x)$.

(E) Inverses as Transformations

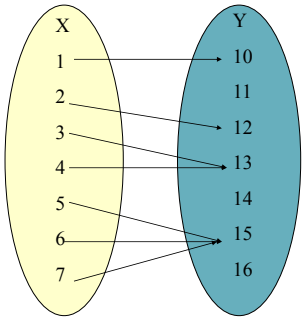
- Place the ordered pairs on the Cartesian plane and we see the relationship between the original ordered pairs and the transformed ordered pairs of the inverse

- The relationship that exists is that the original points are reflected in the line y = x.

(F) Examples

Determine:

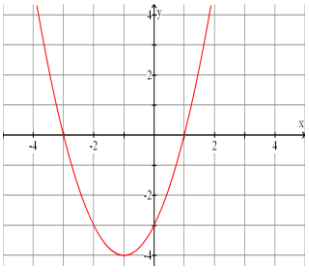
(a) Domain of f(x)
(b) Range of f(x)
(c) Mapping of inverse
(d) Domain of inverse
(e) Range of inverse
(f) Is the inverse a function?



(F) Examples

- Consider a graph of the following data:
- 1. State do domain and range of f
- 2. Evaluate f(-2), f(0), $f^{-1}(1)$, $f^{-1}(-2)$
- 3. Graph the inverse
- 4. Is the inverse a function?
- 5. State the domain and range of $f^{-1}(x)$

- Here is the graph of f(x)



(F) Examples

- Ex. Determine the equation for the inverse of $y = 4x - 9$. Draw both graphs and find the D and R of each.
- Ex. Determine the equation for the inverse function of $y = 2x^2 + 4$. Draw both and find D and R of each.
- Ex. Determine the equation of the inverse function of $y = x^2 + 4x - 5$
- Ex. Determine the equation for the inverse of $y = 2 - \sqrt{(x+3)}$. Draw both and find the domain and range of each.
- Ex. Determine the equation of the inverse of $y = 1/(2x + 3)$. Draw both and find the domain and range of each.

(F) Examples

- ex . The cost of renting a car for a day is a flat rate of \$40 plus \$0.10/km
- 1. Write a function r(d) to represent the total cost of a one day rental. State restrictions on domain and range.
- 2. Find the equation of the inverse. What does the equation of the inverse represent?
- 3. Give an example of how the inverse could be used?

(F) Examples

- ex . If an object is dropped from a height of 80 m, its height above the ground in meters is given by
- $h(t) = -5t^2 + 80$
- 1. Graph the function
- 2. Find and graph the inverse
- 3. Is the inverse a function
- 4. What does the inverse represent?
- 5. After what time is the object 35 m above the ground?
- 6. How long does the object take to fall?

(G) Composing with Inverses

- Let $f(x) = 2x - 7$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph
- Graph the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $f \circ f^{-1}(x)$ and $f^{-1} \circ f(x)$. What do you notice?

(G) Composing with Inverses

- Now let $f(x) = x^2 + 2$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph in a square view window

- Graph the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?

- Now compose as follows $fof^{-1}(x)$ and $f^{-1}of(x)$. What do you notice?

Question 1

- From the following list of ordered pairs, determine:
- G={(-1,2), (0,4), (1,2), (2,7),(3,-1), (5,3)}

- (a) the domain and range of the relation
- (b) the ordered pairs of the inverse relation
- (c) the domain and range of the inverse relation
- (d) Is the inverse relation a function?

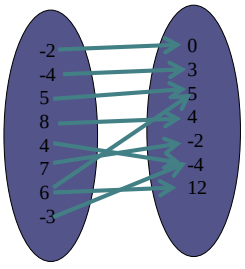
- (e) Evaluate gog(5)
- (f) Evaluate g(3)
- (g) Solve $g(x) = 4$
- (h) Evaluate $g^{-1}(2)$
- (i) Solve $g^{-1}(x) = 5$

Question 2

- From the mapping diagram, determine: •

- (a) the domain and range of the relation
- (b) draw a mapping of the inverse relation
- (c) the domain and range of the inverse relation
- (d) Is the inverse relation a function?

- (e) Evaluate gog(8)
- (f) Evaluate g(-3)
- (g) Solve $g(x) = 4$
- (h) Evaluate $g^{-1}(-4)$
- (i) Solve $g^{-1}(x) = 5$

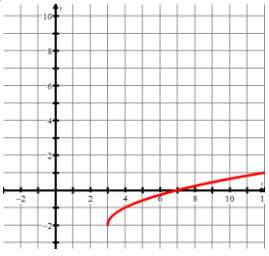


Question 3

- Given the function of $y = f(x)$, determine

- (a) the domain and range of the relation
- (b) the domain and range of the inverse relation
- (c) Is the inverse relation a function?

- (d) Evaluate $fof^{-1}(8)$
- (e) Evaluate f(4)
- (f) Solve $f(x) = ½$
- (g) Evaluate $f^{-1}(1)$
- (h) Solve $f^{-1}(x) = 7$



Question 4

- Given the function of , determine $f(x) = 1 - \sqrt{x-2}$

- (a) the domain and range of the relation
- (b) the domain and range of the inverse relation
- (c) Determine the equation of the inverse relation
- (d) Is the inverse relation a function?

- (e) Evaluate $fof^{-1}(5)$
- (f) Evaluate f(6)
- (g) Solve $f(x) = -2$
- (h) Evaluate $f^{-1}(-3)$
- (i) Solve $f^{-1}(x) = 11$

(H) Internet Links

- Inverse Function Definition - Interactive Applet from AnalyzeMath
- Inverse Function - Interactive Tutorial from AnalyzeMath
- Inverse Functions Lesson - I from PurpleMath

(H) Homework

- HW

- Ex 1F #1,2c, 4bcf, 5,6, 7, 11, 12,13;
- Ex 1G #2, 3