

# Lesson 13 – Rational Functions

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## Lesson Objectives

- ▶ Introduce inverse variation numerically
- ▶ Graph the reciprocal of linear functions and state their properties
- ▶ Study the reciprocal of linear functions in the form of
 
$$f(x) = \frac{a}{b(x+c)} + d \quad \text{and} \quad f(x) = \frac{mx+n}{px+q}$$
- ▶ Graph the reciprocal of quadratic functions and state their properties

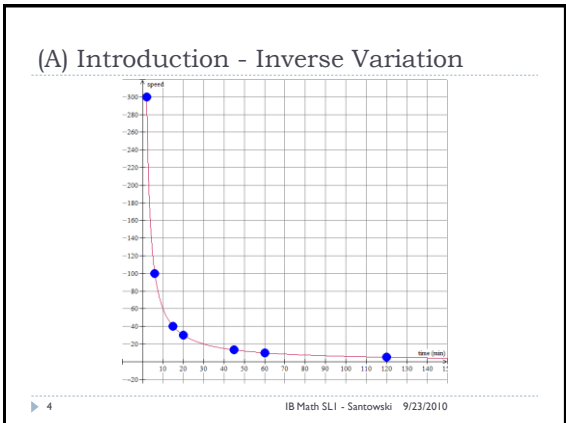
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### (A) Introduction - Inverse Variation

- ▶ The speed at which an object travels 10 km **varies inversely** with the time it takes to travel the 10 kilometers
- ▶ Graph the relation
- ▶ Write an equation for this inverse variation

	Speed	Time
Japanese bullet train	300 km/hr	2 minutes
Mr S. Starex on the SLEX	100 km/hr	6 minutes
Mr S Starex on EDSA	30 km/hr	20 minutes
Mr S running a 10K race	13.3 km/hr	45 minutes
Mr S running a 10 K race now	10 km/hr	60 minutes
My children walking	5 km/hr	120 minutes

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### (B) The Basic Rational Function $f(x) = 1/x$

x	y
-5.00000	-0.20000
-4.00000	-0.25000
-3.00000	-0.33333
-2.00000	-0.50000
-1.00000	-1.00000
0.00000	undefined
1.00000	1.00000
2.00000	0.50000
3.00000	0.33333
4.00000	0.25000
5.00000	0.20000

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### (B) The Basic Rational Function $f(x) = 1/x$

- ▶ The domain of the function is:
- ▶ The range of the function is:
- ▶ The intercepts of the function are:
- ▶ The asymptotes of the function are:

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**(B) The Basic Rational Function**

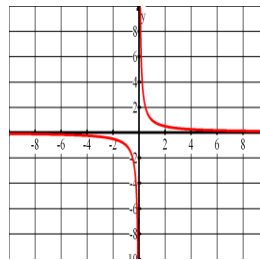
- Notice that ...
- the graphs don't cross either axis; therefore  $f(x) = 1/x$  does not have  $x$  - or  $y$  - intercepts.
- as  $x$  gets very small (close to zero),  $y$  gets very big ( $y$  approaches positive or negative infinity)
- as  $x$  gets very big (as  $x$  approaches positive or negative infinity),  $y$  gets very small (close to zero).
- The basic rational function  $y = 1/x$  may be considered the "Reciprocal Function", since every output  $y$  - value is the reciprocal of the input  $x$  - value (and vice versa).

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**(B) The Basic Rational Function**

▶ Use the graph of  $f(x) = 1/x$  to analyze the END BEHAVIOUR:



- (a) as  $x \rightarrow \infty^+$ , then  $y \rightarrow$
- (b) as  $x \rightarrow \infty^-$ , then  $y \rightarrow$
- (c) as  $x \rightarrow 0^+$ , then  $y \rightarrow$
- (d) as  $x \rightarrow 0^-$ , then  $y \rightarrow$

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**(C) The Rational Function**

$$f(x) = \frac{a}{b(x+c)} + d$$

- ▶ Graph the function  $g(x) = 2x - 5$  and then on the same grid, graph its reciprocal function  $f(x) = \frac{1}{2x-5}$
- ▶ State the domain, range, asymptotes, intercepts of  $y = f(x)$  as well as the transformations of  $y = 1/x$  that moved it to

$$f(x) = \frac{1}{2x-5}$$

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**(C) The Rational Function**

$$f(x) = \frac{a}{b(x+c)} + d$$

- ▶ Work on the following examples → For the rational function, you will determine:
  - (a) equations of asymptotes
  - (b) two points on the fcn
  - (c) intercepts
  - (d) domain and range

(i) Graph  $y = 4 - 2x$  and  $f(x) = \frac{3}{4-2x} \Rightarrow$  connection being???

(ii) Graph  $y = 2x - 6$  and  $g(x) = -4 + \frac{1}{2x-6} \Rightarrow$  connection being???

(iii) Graph  $h(x) = \frac{x+2}{x-1}$  and rewrite in the form of  $h(x) = d + \frac{a}{x-1}$

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**(C) The Rational Function**

$$f(x) = \frac{mx+n}{px+q}$$

- ▶ Show that the function  $f(x) = \frac{4x-1}{2x+1} = 2 + \frac{-3}{2x+1}$
- ▶ State the domain, range, asymptotes, intercepts of  $y = f(x)$  as well as the transformations of  $y = 1/x$  that moved it to

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**(D) Inverse Squared Variations**

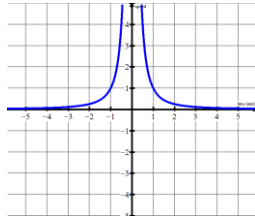
- ▶ Examples → the intensity of light varies inversely as the square of the distance travelled.
- ▶ Examples → the force of gravitational attraction between two bodies in space varies inversely as the square of the distance between them.
- ▶ Graph the function →  $f(x) = 1/x^2$

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(E) The Basic Rational Function  $f(x) = 1/x^2$ 

x	y
-5.00000	0.04000
-4.00000	0.06125
-3.00000	0.11111
-2.00000	0.25000
-1.00000	1.00000
0.00000	undefined
1.00000	1.00000
2.00000	0.25000
3.00000	0.11111
4.00000	0.06125
5.00000	0.04000



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(B) The Basic Rational Function  $f(x) = 1/x^2$ 

- ▶ The domain of the function is:
- ▶ The range of the function is:
- ▶ The intercepts of the function are:
- ▶ The asymptotes of the function are:

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## (C) The Rational Function

$$f(x) = \frac{a}{b(x+c)^2}$$

- ▶ Graph the function  $g(x) = 2x^2 - 8$  and then on the same grid, graph its reciprocal function  $f(x) = \frac{1}{2x^2 - 8}$
- ▶ State the domain, range, asymptotes, intercepts of  $y = f(x)$  as well as the transformations of  $y = 1/x^2$  that moved it to

$$f(x) = \frac{1}{2x^2 - 8}$$

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## (C) The Rational Function

$$f(x) = \frac{a}{b(x+c)^2}$$

- ▶ Graph the function  $g(x) = x^2 - 2x - 8$  and then on the same grid, graph its reciprocal function  $f(x) = \frac{1}{x^2 - 2x - 8}$
- ▶ State the domain, range, asymptotes, intercepts of  $y = f(x)$  as well as the transformations of  $y = 1/x^2$  that moved it to

$$f(x) = \frac{1}{x^2 - 2x - 8}$$

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## (C) The Rational Function

$$f(x) = \frac{a}{b(x+c)^2}$$

- ▶ PREDICT how many & the equations of VAs the function

$$f(x) = \frac{1}{x^2 - 2x + 8}$$

- ▶ State the domain, range, asymptotes, intercepts of  $y = f(x)$  as well as the transformations of  $y = 1/x^2$  that moved it to

$$f(x) = \frac{1}{x^2 - 2x + 8}$$

- ▶ Now graph  $g(x) = x^2 - 2x + 8$  and then on the same grid, graph its reciprocal function

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## (C) The Rational Function

$$f(x) = \frac{a}{b(x+c)^2}$$

- ▶ PREDICT how many & the equations of VAs the function

$$f(x) = \frac{1}{x^2 - 8x + 16}$$

- ▶ State the domain, range, asymptotes, intercepts of  $y = f(x)$  as well as the transformations of  $y = 1/x^2$  that moved it to

$$f(x) = \frac{1}{x^2 - 8x + 16}$$

- ▶ Now graph  $g(x) = x^2 - 8x + 16$  and then on the same grid, graph its reciprocal function

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## (F) Exercises

- ▶ For each of the following, determine (where possible)
- ▶ a) the domain and range
- ▶ b) the coordinates of the x – and y – intercepts
- ▶ c) the equation(s) of the vertical asymptote(s)

(a)  $y = \frac{1}{x}$

(b)  $y = \frac{1}{x-3}$

(c)  $y = -\frac{4}{x-3}$

(d)  $y = \frac{2}{x-4}$

(e)  $y = \frac{x-2}{x+1}$

(f)  $y = \frac{3-2x}{5-3x}$

(g)  $y = \frac{2}{x^2-3x-4}$

(h)  $y = \frac{x+3}{x^2-3x-4}$

(i)  $y = -\frac{1-x^2}{x^2}$

(j)  $y = \frac{1}{x^2}$

(k)  $y = \frac{6}{4-2x^2}$

(l)  $y = \frac{x^2+1}{x^2+2}$

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## HW

- ▶ <http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Graphing%20Simple%20Rational%20Functions.pdf>
- ▶ <http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Graphing%20Rational%20Functions.pdf>

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## Resources

- ▶ Video link #1  
<http://www.onlinemathlearning.com/reciprocal-of-function.html>
- ▶ <http://www.youtube.com/watch?v=8Tyfn3vhjrw>
- ▶ <http://www.youtube.com/watch?v=dCN3YknuLVg>
- ▶

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