

Lesson 21 – The Logarithmic Function – An Inverse Perspective

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Lesson Objectives

- Introduce the function that represents the inverse relation of an exponential function from a graphic and numeric perspective
- Introduce logarithms from an algebraic perspective
- Apply the basic algebraic equivalence of exponential and logarithmic equations to evaluating/simplifying/solving simple logarithmic equations/expressions.

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Terminology (to clarify ... I hope)

- In algebra, the terms EXPONENT and POWER unfortunately are used interchangeably, leading to confusion.
- We will exclusively refer to the number that the base is raised to AS THE EXPONENT and NOT THE POWER.
- For the statement that $2^3 = 8$,
 - a) the base is 2: the base is the number that is repeatedly multiplied by itself.
 - b) the exponent is 3: the exponent is the number of times that the base is multiplied by itself.
 - c) the power is 8: the power is the ANSWER of the base raised to an exponent, or the product of repeatedly multiplying the base by itself an exponent number of times.

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(A) Graph of Exponential Functions

- Graph the exponential function $f(x) = 2^x$ by making a table of values
- What does the input/domain of the function represent?
- What does the output/range of the function represent?
- What are the key graphical features of the function $f(x) = 2^x$?

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(A) Table of Values for Exponential Functions

- | x | y |
|----------|-----------|
| -5.00000 | 0.03125 |
| -4.00000 | 0.06250 |
| -3.00000 | 0.12500 |
| -2.00000 | 0.25000 |
| -1.00000 | 0.50000 |
| 0.00000 | 1.00000 |
| 1.00000 | 2.00000 |
| 2.00000 | 4.00000 |
| 3.00000 | 8.00000 |
| 4.00000 | 16.00000 |
| 5.00000 | 32.00000 |
| 6.00000 | 64.00000 |
| 7.00000 | 128.00000 |
- what does $f(-2) = \frac{1}{4}$ mean $\rightarrow 2^{-2} = \frac{1}{4}$
 - Domain/input \rightarrow the value of the exponent to which the base is raised
 - Range/output \rightarrow the result of raising the base to the specific exponent (i.e. the power)
 - Graphical features \rightarrow y-intercept (0, 1); asymptote; curve always increases

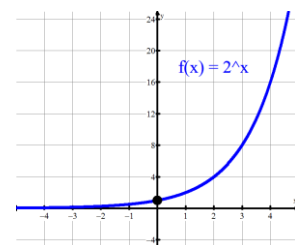
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(A) Graph of Exponential Functions

- | x | y |
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(B) Inverse of Exponential Functions

- List the ordered pairs of the inverse function and then graph the inverse
- Let's call the inverse $I(x)$ for now → so what does $I(1/4)$ mean and equal → $I(1/4) = -2$
- of course $I(x) = f^{-1}(x)$, so what am I asking if I write $f^{-1}(1/4) =$????
- After seeing the graph, we can analyze the features of the graph of the logarithmic function

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(B) Table of Values for the Inverse Function

- | x | y |
|-----------|----------|
| 0.03125 | -5.00000 |
| 0.06250 | -4.00000 |
| 0.12500 | -3.00000 |
| 0.25000 | -2.00000 |
| 0.50000 | -1.00000 |
| 1.00000 | 0.00000 |
| 2.00000 | 1.00000 |
| 4.00000 | 2.00000 |
| 8.00000 | 3.00000 |
| 16.00000 | 4.00000 |
| 32.00000 | 5.00000 |
| 64.00000 | 6.00000 |
| 128.00000 | 7.00000 |
- what does $f^{-1}(1/4) = -2$ mean
 - $2^{-2} = 1/4$
 - Domain/input → the power
 - Range/output → the value of the exponent to which the base is raised that produced the power
 - Graphical features → x-intercept (1,0); asymptote; curve always increases

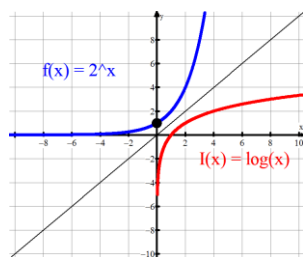
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(B) Table of Values & Graphs for the Inverse Function

x	y
0.03125	-5.00000
0.06250	-4.00000
0.12500	-3.00000
0.25000	-2.00000
0.50000	-1.00000
1.00000	0.00000
2.00000	1.00000
4.00000	2.00000
8.00000	3.00000
16.00000	4.00000
32.00000	5.00000
64.00000	6.00000
128.00000	7.00000



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(C) The Logarithmic Function

- If the inverse is $f^{-1}(x)$ so what I am really asking for if I write $f^{-1}(1/32) = -5$
- The equation of the inverse can be written as $x = 2^y$.
- But we would like to write the equation EXPLICITLY (having the y isolated → having the exponent isolated)
- This inverse is called a logarithm and is written as $y = \log_2(x)$

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(C) Terminology (to clarify ... I hope)

- So what's the deal with the terminology ???
- Given that $f(x) = 2^x$, I can write $f(5) = 2^5 = 32$ and what is meant is that the **EXPONENT 5** is "applied" to the **BASE 2**, resulting in 2 multiplied by itself 5 times ($2 \times 2 \times 2 \times 2 \times 2$) giving us the result of the **POWER of 32**
- The inverse of the exponential is now called a logarithm and is written as $y = \log_2(x)$ or in our case $5 = \log_2(32)$ and what is meant now is that I am taking the **logarithm of the POWER 32** (while working in **BASE 2**) and I get an **EXPONENT of 5** as a result!

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(C) Terminology (to clarify ... I hope)

- So the conclusion → To PRODUCE the POWER, I take a BASE and exponentiate the base a given number of times (ie. The EXPONENT) → this is the idea of an exponential function
- With a logarithmic function → I start with a given POWER and determine the number of times the BASE was exponentiated (i.e. the EXPONENT)
- In math, we use shortcut notations ALL THE TIME. The mathematical shorthand for "What is the exponent on b (the base) that yields p (the power)?" is read as $\log_b p$.

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(D) Playing With Numbers

- Write the following equations in logarithmic form:
 - a) $2^3 = 8$ b) $\sqrt{9} = 3$
 - c) $125^{1/3} = 5$ d) $4^{-1/2} = \frac{1}{2}$
- Write the following equations in exponential form:
 - a) $\log_5 25 = 2$ b) $\log_2(1/16) = -4$
 - c) $\log_4 2 = 0.5$ d) $\log_7 1 = 0$
- Evaluate the following:
 - a) $\log_3(1/27)$ b) $\log_4 8$ c) $\log_{1/3} 4$ d) $\log_3 27 + \log_3 81$

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(D) Common Logarithms

- A common logarithm is a logarithm that uses base 10. You can ignore writing the base in this case: $\log_{10} p = \log p$.
- Interpret and evaluate the following:
 - a) $\log 10$ b) $\log 100$ c) $\log 1000$
 - d) $\log 1$ e) $\log(1/10000)$ f) $\log 1/\sqrt{10}$
- Evaluate the following with your calculator and write the value to three decimal places).
 - a) $\log 9$ b) $\log 10$ c) $\log 11$ d) $\log \pi$

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(D) Working With Logarithms

- Solve: (HINT: switch to exponential form)
- (a) $\log_x 27 = 3$ (b) $\log_x \sqrt[3]{25} = 5$
- (c) $\log_3 8 = \frac{3}{4}$ (d) $\log_x 25 = 2/3$
- (e) $\log_4 \sqrt{2} = x$ (f) $\log_2 27 = x$
- (g) $5 \log_3 9 = x$ (h) $\log_x x = -3$
- (i) $\log_a x = -1.5$ (j) $\log_2(x+4) = 5$
- (k) $\log_3(x-3) = 3$ (l) $\log_2(x^2-x) = \log_2 12$
- (m) $\log_3 \sqrt[5]{9} = x$ (n) $\log_{1/3} \sqrt[9]{27} = x$
- (o) $\log_5 81 = -4$ (p) $\log_2 \sqrt{0.125} = x$

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Further Examples

- <http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Meaning%20of%20Logarithms.pdf>
- <http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Graphing%20Logarithms.pdf>
- <http://www.ma.utexas.edu/users/asalerno/worksheet12.pdf>
- <http://www.ma.utexas.edu/users/asalerno/worksheet13.pdf>

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(D) Working With Logarithms

- Evaluate $\log_3 9$ and $\log_3 3$
- Evaluate $\log_5 125$ and $\log_{125} 5$
- What relationship exists between the values of $\log_a b$ and $\log_b a$
- Solve the system defined by $2^x + y = 32$ and $2^x \cdot y = 8$
- Solve $\log_7(\log_4 x) = 0$
- Solve $\log_5(\log_2(\log_3 x)) = 0$

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(E) Transformed Logarithmic Functions

- As will be seen in the next exercises, the graph maintains the same "shape" or characteristics when transformed
- Depending on the transformations, the various key features (domain, range, intercepts, asymptotes) will change

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(E) Transformed Logarithmic Functions

- So we can now do a complete graphic analysis of this graph
- (i) no y-intercept and the x-intercept is 1
- (ii) the y axis is an asymptote
- (iii) range $\{y \in \mathbb{R}\}$
- (iv) domain $\{x > 0\}$
- (v) it increases over its domain
- (vi) it has no max/min or turning points

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(E) Graphing Log Functions

- Without using graphing technology, graph the following functions (it may help to recall your knowledge of function transformations)
- (1) $f(x) = \log_2(x + 2)$
- (2) $f(x) = -3\log_2(x - 4)$
- (3) $f(x) = \log_5(4x - 4) + 5$
- Examples and discussions on how to make these graphs is found at the following website:
- [Graphs of Logarithmic Functions from AnalyzeMath](#)

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(F) Homework

- From PP12 Math Textbook, S10.2, p394-396;
- Q3-5, 7col, 8col, 9, 10, 12-14

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