Jun. 2

Applying Derivatives - Introduction to Optimization

Problems that ask for maximum or minimum values are called optimization problems, and each solution usually requires a multi-step procedure. This procedure can often be summarized as follows:

1. Read the problem carefully and note which quantities vary and which are constant.
2. Draw a diagram. Label the components with appropriate variables and constants.
3. Identify the quantity to be maximized or minimized and write a function for it.
4. Find the domain of the function in the problem.
5. Graph the function on the domain in Step 4.
6. Identify critical points and any endpoints. The critical points and endpoints are the candidates for any maximum and minimum values of the function.
7. Use the graph, the first derivative, and perhaps the second derivative to find the maximum and/or minimum values.
8. Answer the original question. Does your answer make sense? Is it justified?

It is easy to get involved in the individual steps and lose sight of this lengthy overall process. A graphing calculator can help you focus on the overall process by assisting with the individual steps. A graphing calculator can also make it possible to do more difficult problems that could not be done without using technology.

The Box Problem

You will start with a rectangular piece of 8.5 x 11 cm sheet of paper, from which you will make an open topped box using the following instructions ➔ you will cut out 4 squares from each corner of the rectangular sheet and then fold up the sides to complete the box. The size of the square that you will cut out will vary from student to student as we create a class set of data.

(a) From your sheet of A4 paper, measure the squares that you have been assigned. Cut out the square that you marked out in part (a).
(b) CAREFULLY, fold the sides into a box and tape together the edges to complete the lateral faces of the box.
(c) Measure the length, width and height of your box. Record these values.
(d) Calculate your box's volume.
(e) Record the size of your square and calculated volume on the class data table.
(f) Create a scatter-plot of the class data and use regression to determine the equation of the function that best fits the data and the CONTEXT of the data.
(g) Use the graph to determine what size of a square should be cut out from the A4 paper in order to maximize the volume of the box. What are the dimensions on the box that produce this optimal volume?

(h) Now, incorporate some basic algebra. Review the process by which you constructed the box in the first place. Show how to develop a function for the volume, so that the volume is a function of the size of the square removed from the corners, i.e. V(x) = ????
(i) Now, incorporate basic calculus. What do you do with calculus in order to find the maximums on the V(x) function?
(j) So, now apply the calculus in order to "check" if the volume and size of square from our class data are in fact correct.
(k) So to make sure you really understand the process, from me you will get dimensions for another sheet of paper, then develop a formula, apply the calculus and find what size of square I need to cut out in order to optimize the volume of the box.

The Cone Problem – Part 1

Removing a sector from the circular piece of paper and fastening together the remaining seams creates a cone. The following activity will lead you to finding the angle of the removed sector that produces a cone of maximum volume. The maximum volume will also be found. We will work together as a class to gather data to prepare a scatter-plot of the results, from which we can determine the angle of the sector that results in the construction of a cone that maximizes the volume of the cone.

(a) From your handout of the circle, measure the central angle that you have been assigned. Draw the sector upon your circle.
(b) Cut out the sector that you marked out in part (a). KEEP this sector!!!
(c) Use a piece of thread/string to measure the radius and arc length of BOTH sectors. Record these values.
(d) CAREFULLY, roll the sector into a cone and tape together the 2 "sides" to complete the lateral face of the cone.
(e) Find some way to determine the HEIGHT and RADIUS of the CONE. Record these values.
(f) Calculate your cone's height.
(g) Record your sector angle and calculated volume on the class data table.
(h) Create a scatter-plot of the class data and draw in a smooth, best fitting curve given the data points.
(i) You can try to use regression to determine the equation of the function that best fits the data and the CONTEXT of the data, but this is not that easy.
(j) Use your scatter-plot to determine what sector angle should be used to create the lateral surface in order to maximize the volume of the cone. What are the dimensions of the cone? What IS this maximum volume?
(l) OPTIONAL:
    a. Now, incorporate some basic algebra. Review the process by which you constructed the cone in the first place. Show how to develop a function for the volume, so that the volume is a function of the angle of the sector removed from the circle, i.e. V(θ) = ???? HINT: can you use θ to determine an expression for the radius of the cone and then for the height of the cone?
    b. Now, incorporate basic calculus. What do you do with calculus in order to find the maximums on the V(x) function?
    c. So, now apply the calculus in order to "check" if the volume and size of angle from our class data are in fact correct.