Differentiation Rules

Constant Rule:	$\frac{d}{dx}(c) = 0$
Constant Multiple Rule:	$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$
Sum Rule:	$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$
Difference Rule:	$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$
Product Rule:	$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$
Quotient Rule:	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}$
Chain Rule:	$\frac{d}{dx}[f(g(x))] = f'(g(x))\frac{d}{dx}g(x)$
Power Rule:	$\frac{d}{dx}(x^n) = nx^{n-1}$
Power and Chain Rules:	$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$

PROBLEMS PLUS

Graph the function $f(x) = |x - 2| + |x - 5|$. Where is f discontinuous? Where is it not differentiable?

EXERCISE 2.6

B **1.** Find the derivatives of the following functions.

(a) $F(x) = (5 - 3x)^7$

(b) $F(x) = (2x^2 + 1)^{20}$

(c) $G(x) = (x^3 + x^2 - 2)^{\frac{3}{4}}$

(d) $G(x) = \sqrt{x^4 - x + 1}$

(e) $y = \sqrt[4]{x^2 + x}$

(f) $y = (1 + 3x + 4x^2)^{-3}$

(g) $y = \dfrac{1}{(x^3 + 2x^2 + 1)^2}$

(h) $y = \dfrac{4}{\sqrt{9 - x^2}}$

(i) $y = (1 + 2\sqrt{x})^6$

(j) $y = \sqrt{x + \sqrt{x}}$

(k) $y = x - \sqrt[5]{1 + x^5 - 6x^{10}}$

(l) $y = x^2 + (x^2 - 1)^5$

2. If $y = u^4 + 5u^2$, where $u = x^5 + 2x^2 + 1$, find $\frac{dy}{dx}$. Leave your answer in terms of u and x.

3. Find $\left.\dfrac{dy}{dx}\right]_{x=4}$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$.

4. Find $\left.\dfrac{dy}{dt}\right]_{t=1}$ if $y = \sqrt{1 + r^2}$ and $r = \dfrac{t + 1}{2t + 1}$.

5. Find $\left.\dfrac{ds}{dt}\right]_{t=4}$ if $s = v + \dfrac{50}{v}$ and $v = 3t - \sqrt{t}$.

6. Differentiate:

(a) $F(x) = x\sqrt{x^2 + 1}$ (b) $F(x) = (2x + 1)(4x - 1)^5$

(c) $G(x) = (x^2 - 1)^4(2 - 3x)$

(d) $G(x) = (x^4 - x + 1)^2(x^2 - 2)^3$

(e) $F(x) = \dfrac{x}{\sqrt{2x + 3}}$ (f) $f(t) = \dfrac{(1 + 2t)^5}{(3t^2 - 5)^2}$

(g) $g(x) = \left(\dfrac{x + 2}{x - 2}\right)^3$ (h) $h(t) = \left(\dfrac{t^2 + 1}{t + 1}\right)^{10}$

(i) $y = \sqrt{\dfrac{x^2 - 1}{x^2 + 1}}$ (j) $y = \dfrac{(2x + 3)^3}{\sqrt{4x - 7}}$

(k) $y = 3\sqrt{x}(2x + 1)^5 + \sqrt{4x - 3}$

(l) $y = \sqrt{1 + \sqrt[3]{x}}$

(m) $y = (t + \sqrt[3]{t + t^2})^{20}$ (n) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

7. Find the equation of the tangent line to the curve $y = (x^2 - 3)^8$ at the point $(2, 1)$.

8. Find the equation of the tangent line to the curve $y = \dfrac{1}{\sqrt{20 - x^4}}$ at the point $\left(2, \frac{1}{2}\right)$.

9. If $F(x) = f(g(x))$, where $g(2) = 4$, $g'(2) = 3$, and $f'(4) = 5$, find $F'(2)$.

10. If $G(x) = h(p(x))$, where $h(5) = 1$, $h'(5) = 2$, $h'(1) = 3$, $p(1) = 5$, and $p'(1) = 7$, find $G'(1)$.

C **11.** If f is a differentiable function, find expressions for the derivatives of the following functions.

(a) $F(x) = f(x^4)$ (b) $G(x) = [f(x)]^4$

(c) $H(x) = f(\sqrt{x})$ (d) $P(x) = \sqrt{f(x)}$

(e) $y = f(f(x))$ (f) $y = \sqrt{1 + [f(x)]^2}$

(g) $y = [f(x^2)]^2$ (h) $y = f([f(x)]^3)$

12. (a) Use the Chain Rule and the fact that $|x| = \sqrt{x^2}$ to show that

$$\frac{d}{dx}|x| = \frac{x}{|x|}$$

(b) Sketch the graphs of the function $f(x) = |x|$ and its derivative.

(c) Use the result of part (a) to differentiate the function $g(x) = x|x|$.

(i) $y' = \dfrac{6(1 + 2\sqrt{x})^5}{\sqrt{x}}$

(j) $y' = \dfrac{2\sqrt{x} + 1}{4\sqrt{x}\sqrt{x + \sqrt{x}}}$

(k) $y' = 1 - \dfrac{x^4 - 12x^9}{(1 + x^5 - 6x^{10})^{\frac{4}{5}}}$

(l) $y' = 2x + 10x(x^2 - 1)^4$

2. $(4u^3 + 10u)(5x^4 + 4x)$ **3.** -117

4. $-\dfrac{2}{9\sqrt{13}}$ **5.** $\frac{11}{8}$

6. (a) $F'(x) = \dfrac{2x^2 + 1}{\sqrt{x^2 + 1}}$

(b) $F'(x) = 6(8x + 3)(4x - 1)^4$

(c) $G'(x) = (3 + 16x - 27x^2)(x^2 - 1)^3$

(d) $G'(x) = 2(x^4 - x + 1)(x^2 - 2)^2 \times (7x^5 - 8x^3 - 4x^2 + 3x + 2)$

(e) $F'(x) = \dfrac{x + 3}{(2x + 3)^{\frac{3}{2}}}$

(f) $f'(t) = \dfrac{2(1 + 2t)^4(3t^2 - 6t - 25)}{(3t^2 - 5)^3}$

(g) $g'(x) = \dfrac{-12(x + 2)^2}{(x - 2)^4}$

(h) $h'(t) = \dfrac{10(t^2 + 1)^9(t^2 + 2t - 1)}{(t + 1)^{11}}$

(i) $y' = \dfrac{2x}{(x^2 + 1)^{\frac{3}{2}}\sqrt{x^2 - 1}}$

(j) $y' = \dfrac{4(2x + 3)^2(5x - 12)}{(4x - 7)^{\frac{3}{2}}}$

(k) $y' = \dfrac{3(2x + 1)^4[22x + 1]}{2\sqrt{x}} + \dfrac{2}{\sqrt{4x - 3}}$

(l) $y' = \dfrac{1}{6x^{\frac{2}{3}}\sqrt{1 + \sqrt[3]{x}}}$

(m) $y' = 20(t + \sqrt[3]{t + t^2})^{19}\left(1 + \dfrac{1 + 2t}{3(t + t^2)^{\frac{2}{3}}}\right)$

(n) $y' = \dfrac{1}{2\sqrt{x + \sqrt{x + \sqrt{x}}}}\left(1 + \dfrac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x + \sqrt{x}}}\right)$

7. $32x - y - 63 = 0$ **8.** $4x - 2y - 7 = 0$

9. 15 **10.** 14

11. (a) $F'(x) = 4x^3f'(x^4)$ (b) $G'(x) = 4[f(x)]^3f'(x)$

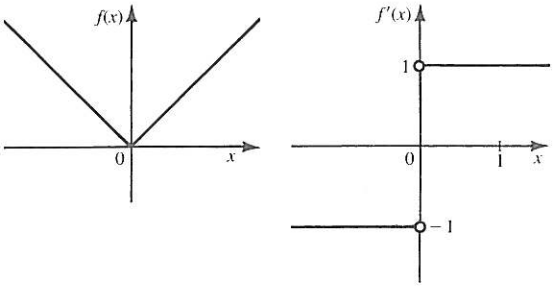
(c) $H'(x) = \dfrac{1}{2\sqrt{x}}f'(\sqrt{x})$

(d) $P'(x) = \dfrac{f'(x)}{2\sqrt{f(x)}}$ (e) $y' = f'(f(x))f'(x)$

(f) $y' = \dfrac{f(x)f'(x)}{\sqrt{1 + [f(x)]^2}}$ (g) $y' = 4xf(x^2)f'(x^2)$

(h) $y' = 3f'(x)[f(x)]^2f'([f(x)]^3)$

12. (b)



(c) $g'(x) = 2|x|$

EXERCISE 2.7

1. (a) $\dfrac{x}{y}$ (b) $-\dfrac{x^2}{y^2}$ (c) $-\dfrac{y}{x}$ (d) $-\dfrac{y + 2x}{x + 2y}$

(e) $\dfrac{2y - x^2}{y^2 - 2x}$ (f) $\dfrac{2x - 2y^2}{4xy - 3y^2}$ (g) $-\dfrac{\sqrt{y}}{\sqrt{x}}$

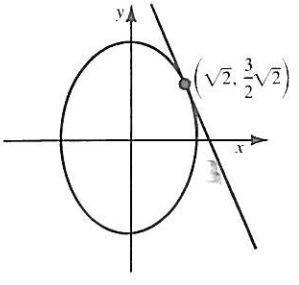
(h) $\dfrac{2y}{(x + y)^2 + 2x}$

2. (a) $\frac{1}{4}$ (b) -8 (c) $\frac{7}{8}$ (d) 3 (e) -1 (f) -4

3. (a) $2x - y + 1 = 0$ (b) $4x + y - 9 = 0$
(c) $3x - 16y + 25 = 0$ (d) $x + y = 0$

4. (a) $-\frac{3}{2}$ (c) $3x + 2y - 6\sqrt{2} = 0$

(d)



5. (a) $3x - 4y - 14 = 0$

(b)