IBSL1 Exponentials Review (past IB questions)

1. A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\,e^{0.4t}$.

(a) How many leopards are there after 2 years?

(b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

(Total 4 marks)

2. \$1000 is invested at 15% per annum interest, **compounded monthly**. Calculate the minimum number of months required for the value of the investment to exceed \$3000.

(Total 6 marks)

3. The mass m kg of a radio-active substance at time t hours is given by

$$m = 4e^{-0.2t}.$$

(a) Write down the initial mass.

(b) The mass is reduced to 1.5 kg. How long does this take?

(Total 6 marks)

4. Find the **exact** value of x i each of the following equations.

(a) $5^{x+1} = 625$

(b) $\log_{\mathbf{a}} (3x + 5) = 2$

(Total 6 marks)

5. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After n years the number of taxis, T, in the city is given by

$$T = 280 \times 1.12^n.$$

(a) (i) Find the number of taxis in the city at the end of 2005.

(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

(b) At the end of 2000 there were 25 600 people in the city who used taxis. After n years the number of people, P, in the city who used taxis is given by

$$P = \frac{2560000}{10+90e^{-0.1n}}.$$

(i) Find the value of P at the end of 2005, giving your answer to the nearest whole number.

(ii) After seven complete years, will the value of P be double its value at the end of 2000? Justify your answer.

(6)

(c) Let R be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if $R < 70$.

(i) Find the value of R at the end of 2000.

(ii) After how many complete years will the city first reduce the number of taxis?

(5)

(Total 17 marks)

Answers/Markscheme

1. (a) At $t = 2$, $N = 10e^{0.4(2)}$ (M1)
$N = 22.3$ (3 sf)
Number of leopards = 22 (A1)

(b) If $N = 100$, then solve $100 = 100e^{0.4t}$
$10 = e04^t$
$\ln 10 = 0.4t$
$t = \frac{\ln 10}{0.4} \sim 5.76$ years (3 sf) (A1)

[4]

2. 15% per annum = $\frac{15}{12}\% = 1.25\%$ per month (M1)(A1)

Total value of investment after n months, $1000(1.0125)^n > 3000$ (M1)
$\Rightarrow (1.0125)^n > 3$

$n \log (1.0125) > \log (3) \Rightarrow n > \frac{\log(3)}{\log(1.0125)}$ (M1)

Whole number of months required so $n = 89$ months. (A1) (C6)

Notes: *Award (C5) for the answer of 90 months obtained from using n – 1 instead of n to set up the equation.*
Award (C2) for the answer 161 months obtained by using simple interest.
Award (C1) for the answer 160 months obtained by using simple interest.

[6]

3. (a) Initial mass $\Rightarrow t = 0$ (A1)
mass = 4 (A1) (C2)

(b) $1.5 = 4e^{-0.2t}$ (or $0.375 = e^{-0.2t}$) (M2)
$\ln 0.375 = -0.2t$ (M1)
$t = 4.90$ hours (A1) (C4)

[6]

4. (a) **METHOD 1**

$5^{x+1} = 5^4$ A1
$x + 1 = 4$ (A1)
$x = 3$ A1 N2

METHOD 2

Taking logs A1

eg $x + 1 = \log_5 625$, $(x + 1)\log 5 = \log 625$

$x + 1 = \frac{\log 625}{\log 5}$ $(x+1=4)$ (A1)

$x = 3$ A1 N2

(b) **METHOD 1**

Attempt to re-arrange equation (M1)

$3x + 5 = a^2$ A1

$x = \frac{a^2 - 5}{3}$ A1 N2

METHOD 2

Change base to give $\log (3x + 5) = \log a^2$ (M1)

$3x + 5 = a^2$ A1

$x = \frac{a^2 - 5}{3}$ A1 N2

[6]

5. (a) (i) $n = 5$ (A1)

$T = 280 \times 1.12^5$

$T = 493$ A1 N2

(ii) evidence of doubling (A1)
eg 560
setting up equation A1
eg $280 \times 1.12^n = 560$, $1.12^n = 2$
$n = 6.116...$ (A1)
in the year 2007 A1 N3

(b) (i) $P = \frac{2560000}{10 + 90\,e^{-0.1(5)}}$ (A1)

$P = 39\,635.993...$ (A1)

$P = 39\,636$ A1 N3

(ii) $P = \frac{2560000}{10 + 90\,e^{-0.1(7)}}$

$P = 46\,806.997...$ A1
not doubled A1 N0
valid reason for **their** answer R1
eg $P < 51200$

(c) (i) correct value A2 N2

$e.g. \frac{25600}{280}, 91.4, 640:7$

(ii) setting up an inequality (accept an equation, or reversed inequality) M1

$e.g. \frac{P}{T} < 70, \frac{2560000}{\left(10 + 90e^{-0.1n}\right)280 \times 1.12^n} < 70$

finding the value 9.31.... (A1)

after 10 years A1 N2

[17]