Name: ____________________

SL – Logarithms Quiz

Attempt **all** questions. Time allowed is 30 minutes. [30 points total]

Q1. Simplify the following (i.e. express without logs) if $\log 2 = a$, $\log 3 = b$ and $\log 5 = c$. [3 each]	
$\log 25 - \log \sqrt{3}$	$\log 45 + \log \frac{9}{16}$

Q2. Simplify the following, and then find their values, to 3 s.f. [3 each]	
$\log_7 5 - \log_7 10$	$2\log_e 3 + 3\log_e 4$

Q3. An investor wants to raise $10,000. She deposits $2,000 in an account earning 10% compounded annually. How many years will she have to wait? [4]

Q4. The amount of radioactive material that is left is governed by the equation $A = 40e^{-0.7t}$, where A is measured in grams and t is measured in days.

a) What is the half-life of the material, that is, how long will it take for the material to reach half of its original mass? [4]
b) How long will it take for there to be only 10 grams of radioactive material left? [2]

Q5. Find the equation of the asymptote and the coordinate y-intercept of $y = -e^{-2x} - 3$. [1 each]

equation of asymptote	**coordinate** of y-intercept

Q6. Simplify then evaluate the following. [3 each]

$\frac{\log_4 27}{\log_4 81}$	$\frac{\log_7 \sqrt{7}}{\log_{10} 1000}$

SL Logarithms Quiz Solutions and Feedback

Since I am not at school to personally deliver feedback about the logarithm quizzes, and see similar misunderstandings among many students, I hope that this document will be helpful to you in preparing for the test on Tuesday.

1. Simplify the following (i.e. express without logs) if $\log 2 = a$, $\log 3 = b$, and $\log 5 = c$.

According to the given values of *a*, *b*, and *c*, the strategy is to break down each number into products and quotients of powers with bases 2, 3, and 5. Most students were successful at this. However, the common mistake that followed was to assume (for example) that $\log 5^2 = c^2$, which is incorrect. $(\log 5)^2 = c^2$, but $(\log 5)^2$ is not the same thing as $\log 5^2$, because in the second expression, the logarithm is not part of the quantity being squared. The correct approach would be to use the logarithmic law to move the exponents down as coefficients (see highlighted steps below). Only then can the logarithmic expressions be correctly substituted for *a*, *b*, and *c*. There was also a common silly arithmetic error of $3^3 = 9$: double-check your powers carefully! There was a frightening number of students who wrote things like $c^2 - b^{\frac{1}{2}} = \frac{c^2}{b^{\frac{1}{2}}}$ . . . does this mean that $8 - 2 = \frac{8}{2}$?!! Only use logarithmic laws WHEN THERE ARE LOGARITHMS still remaining in the expressions!

$$\log 25 - \log\sqrt{3}$$
$$= \log 5^2 - \log 3^{\frac{1}{2}}$$
$$= 2\log 5 - \frac{1}{2}\log 3$$
$$= 2c - \frac{1}{2}b$$
$$= -\frac{1}{2}b + 2c$$

$$\log 45 - \log\frac{9}{16}$$
$$= \log 3^2 \cdot 5 - \log\frac{3^2}{2^4}$$
$$= \log 3^2 + \log 5 + \log 3^2 - \log 2^4$$
$$= 2\log 3 + \log 5 + 2\log 3 - 4\log 2$$
$$= -4\log 2 + 4\log 3 + \log 5$$
$$= -4a + 4b + c$$

2. Simplify the following, and then find their values to 3 s.f.

Simplifying a logarithmic expression means to express it with as few logarithms as possible – usually, it can be simplified to just a single logarithm. There were a lot of careless errors with students leaving out the bases of 7 or *e* as they simplified, which meant that they evaluated their simplified expressions using logarithms with a base of 10 instead of base 7 and base *e*. There was also a common silly arithmetic error of $\frac{5}{10} = 2$. The worst mistake was $\log_7 5 - \log_7 10 = \frac{\log_7 5}{\log_7 10}$, followed by the even more frightening belief that the $\log_7$ could then be cancelled on the top and bottom, leaving an answer of $\frac{5}{10}$. Some students jumped straight into the change-of-base property which meant that they didn't successfully simplify the logarithmic expressions before evaluating. To simplify properly, move the coefficients into the exponents so that the logarithmic product and quotient laws can be used to express as a single logarithm.

$$\begin{aligned} &\log_7 5 - \log_7 10 \\ &= \log_7\left(\frac{5}{10}\right) \\ &= \log_7 0.5 \end{aligned}$$

$$\begin{aligned} &2\log_e 3 + 3\log_e 4 \\ &= \log_e 3^2 + \log_e 4^3 \\ &= \log_e 9 + \log_e 64 \\ &= \log_e (9 \cdot 64) \\ &= \log_e 576 \end{aligned}$$

Now that each of the expressions has been simplified, they are ready to be evaluated. Note that the base of 7 in the first problem can be quickly resolved with a change of base. It is important to round the final answers to THREE significant figures.

$$\begin{aligned} &\log_7 0.5 \\ &= \frac{\log 0.5}{\log 7} \\ &\approx -0.356 \end{aligned}$$

$$\begin{aligned} &\log_e 576 \\ &= \ln 576 \\ &\approx 6.36 \end{aligned}$$

3. An investor wants to raise $10,000. She deposits $2,000 in an account earning 10% compounded annually. How many years will she have to wait?

This question was solved quite well by most students if they knew the compound interest formula. Since the compound interest formula does not appear in the IB formula booklets, it is very important that it is memorized accurately. Many students did not correctly interpret the word "annually" as compounding just once each year – in other words, $n = 1$. This means that the general compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ becomes the much simpler annual compound interest formula $A = P(1 + r)^t$. A common silly mistake was to write the annual interest rate of 10% as 0.01 instead of 0.1.

$$10000 = 2000(1 + 0.1)^t$$
$$\frac{10000}{2000} = (1 + 0.1)^t$$
$$5 = 1.1^t$$
$$\log_{1.1} 5 = t$$
$$t = \frac{\log 5}{\log 1.1}$$
$$t \approx 16.9 \text{ years}$$

Note: Although full credit was given for this answer since it was rounded to three significant figures, the actual correct answer is 17 years. Because the money is compounded just once each year, the investor must wait until the end of the 17^{th} year before the money is finally compounded to bring the total amount above $10,000. After 16.9 years, the amount of accumulated money will be the same as it was after it was compounded at the end of the 16^{th} year, and the investor won't yet have the $10,000 that she wants to raise.

4. The amount of radioactive material that is left is governed by the equation $A = 40e^{-0.7t}$, where *A* is measured in grams and *t* is measured in days.

a) What is the half-life of the material, that is, how long will it take for the material to reach half of its original mass?

Many students didn't understand what was meant by "half of its original mass", and put down $\frac{1}{2}$ as the value of *A* in the equation. The original mass occurs when $t = 0$, meaning that half of the original mass is $\frac{1}{2}\left(40e^{-0.7(0)}\right) = \frac{1}{2}(40) = 20$ grams. Other students failed to divide both sides of the equation by 40 so that they could isolate the power $e^{-0.7t}$ before taking the logarithm of both sides to solve for the exponent. Another common error was not giving answers to three significant figures – time is continuous, and nowhere does the problem stipulate that there must be an integer number of days that have elapsed.

$$20 = 40e^{-0.7t}$$
$$0.5 = e^{-0.7t}$$
$$\ln 0.5 = \ln e^{-0.7t}$$
$$\ln 0.5 = -0.7t$$
$$t = \frac{\ln 0.5}{-0.7}$$
$$t \approx 0.990 \text{ days}$$

b) How long will it take for there to be only 10 grams of radioactive material left?

Although most students chose to answer this question by substituting 10 grams for *A*, there was another (easier) way to solve. 10 grams is exactly half of the 20 grams that were left in the radioactive material after part a), meaning that another half-life has passed. Since the half-life was found to be 0.990 days in part a), it must take an additional 0.990 days for the substance to decay from 20 grams to 10 grams. Therefore, 0.990 + 0.990 = 1.98 days. Again, the final answer should be rounded to three significant figures.

$$10 = 40e^{-0.7t}$$
$$0.25 = e^{-0.7t}$$
$$\ln 0.25 = \ln e^{-0.7t}$$
$$\ln 0.25 = -0.7t$$
$$t = \frac{\ln 0.25}{-0.7}$$
$$t \approx 1.98 \text{ days}$$

5. Find the equation of the asymptote and the coordinates of the *y*-intercept of $y = -e^{-2x} - 3$.

The asymptote of an exponential function of the form $y = a^x$ is a horizontal line on top of the *x*-axis with equation $y = 0$. The negative sign in front of the given function will reflect the exponential graph vertically across the *x*-axis without changing the horizontal asymptote, but the –3 at the end will shift the entire function down by 3 units. This means that the horizontal asymptote will also be shifted down by 3 units and will have the equation $y = -3$.

At the *y*-intercept, $x = 0$, so $y = -e^{-2(0)} - 3 = -1 - 3 = -4$. Although full credit was given for the answer $y = -4$, the question asks for the coordinates of the *y*-intercept, so the actual answer is $(0,-4)$.

6. Simplify then evaluate the following.

Most students seemed to arrive at the answer to the first problem almost by luck, since they appeared to cancel the $\ln 3$ on the top and bottom of the fraction <u>before</u> moving the exponents into the coefficients. A lot of students made a major mistake by stating that $\frac{\ln 3^3}{\ln 3^4} = \ln 3^3 - \ln 3^4$, which is incorrect. $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$ and not $\frac{\ln a}{\ln b}$, so students must take care not to confuse their logarithmic laws. In the second problem, the best strategy was to express $\sqrt{7}$ and 1000 as powers of 7 and 10 respectively, so that the bases of the powers would cancel with the bases of the logarithms. Some students had difficulty dividing $\frac{1}{2}$ by 3 – it might have been easiest to re-write this division as a multiplication by the reciprocal of 3: $\frac{1}{3}$.

$$\frac{\ln 27}{\ln 81} = \frac{\ln 3^3}{\ln 3^4} = \frac{3\ln 3}{4\ln 3} = \frac{3}{4}$$

$$\frac{\log_7 \sqrt{7}}{\log_{10} 1000} = \frac{\log_7 7^{\frac{1}{2}}}{\log_{10} 10^3} = \frac{\frac{1}{2}}{3} = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$