10.7 Working with Proof

The process of proof occurs over and over again. The concepts and skills you have learned about logarithms are combined with the strategies for solving problems to solve any problem or prove any fact. In order to do this you must understand clearly

A: what you are asked to prove.
B: what information you are given.

Think of the above as you do the proof in the following example.

Example If $\log_x y = m^2$ and $\log_y x = \frac{4}{m}$, prove that $m = \frac{1}{4}$.

Solution $\log_x y = m^2$ suggests $y = x^{m^2}$ ①

$\log_y x = \frac{4}{m}$ suggests $x = y^{\frac{4}{m}}$ ②

Think: What clues are given in the question?

From ① and ② you can obtain

$x = y^{\frac{4}{m}}$

$x = (x^{m^2})^{\frac{4}{m}}$ since $y = x^{m^2}$

$x = x^{4m}$ ③

From ③, since the bases are equal, then the exponents are equal.

$$1 = 4m \text{ or } m = \frac{1}{4}$$

which is what you wanted to prove.

10.7 Exercise

B 1 Given that $\log_m p = 2a^2$ and $a \log_p m = 3$. Prove that $a = \frac{1}{6}$.

2 Prove that if $\log_x y = am^2$, and for all a, b, m, $bam = 1$, then $\log_y x = \frac{b}{m}$.

3 Prove that for all m and n, $\log_m A = \frac{\log_n A}{\log_n m}$.

4 If $a^2 + b^2 = 23ab$, prove that $\log\left(\frac{a+b}{5}\right) = \frac{\log a + \log b}{2}$.

5 If $a(a - 7b) = -b^2$, then prove that $\log\left(\frac{a+b}{3}\right) = \frac{\log a + \log b}{2}$.

6 Prove each of the following for $a, b, \in R$, and $b > 0$.

(a) $\log_p \frac{a}{b} + \log_p b = \log_p a$ (b) $\log_x \frac{a}{b} = \log_x a - \log_x b$