The following definitions then form the basis of trigonometry.

Primary Trigonometric Values

$$\text{sine } \theta = \frac{y}{r} \qquad \text{cosine } \theta = \frac{x}{r} \qquad \text{tangent } \theta = \frac{y}{x}$$

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x}$$

By writing the reciprocals of the above, other trigonometric values are defined.

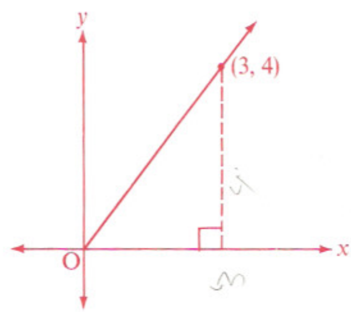
Reciprocal Trigonometric Values

$$\text{cosecant } \theta = \frac{r}{y} \qquad \text{secant } \theta = \frac{r}{x} \qquad \text{cotangent } \theta = \frac{x}{y}$$

$$\csc \theta = \frac{r}{y} \qquad \sec \theta = \frac{r}{x} \qquad \cot \theta = \frac{x}{y}$$

To calculate the trigonometric values you need find only a point on the terminal arm.

Example 1 The point (3, 4) is on the terminal arm of angle θ as shown. Calculate the trigonometric values.



Solution From the diagram $r = \sqrt{x^2 + y^2}$

$$= \sqrt{(3)^2 + (4)^2}$$
$$= 5.$$

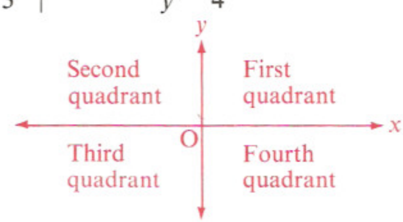
Use $x = 3$, $y = 4$, $r = 5$.

$$\sin \theta = \frac{y}{r} = \frac{4}{5} \qquad \cos \theta = \frac{x}{r} = \frac{3}{5} \qquad \tan \theta = \frac{y}{x} = \frac{4}{3}$$

$$\csc \theta = \frac{r}{y} = \frac{5}{4} \qquad \sec \theta = \frac{r}{x} = \frac{5}{3} \qquad \cot \theta = \frac{x}{y} = \frac{3}{4}$$

Trigonometric values are often expressed in fractional form.

In the next example, the terminal arm of the angle occurs in the second quadrant.



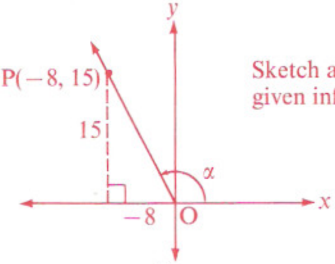
Example 2 P(−8, 15) is a point on the terminal arm of angle α. Calculate its trigonometric values.

Solution From the diagram,

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-8)^2 + (15)^2}$

$r = 17$



Sketch a diagram with the given information.

Use $x = -8$, $y = 15$, $r = 17$

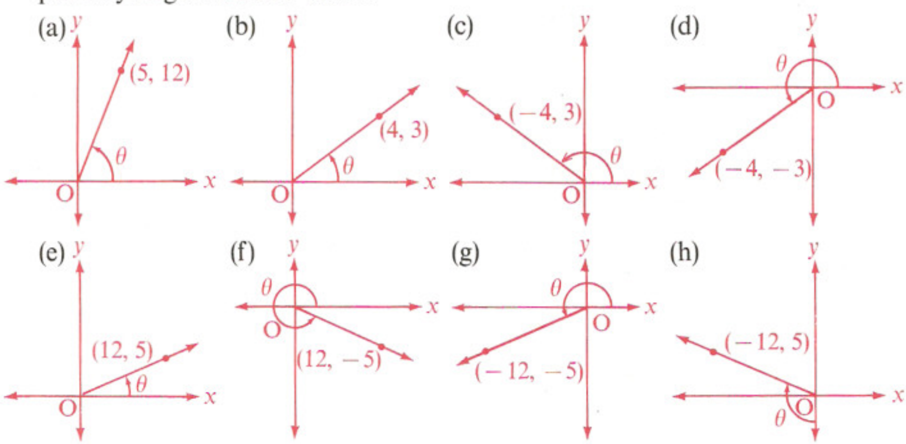
$$\sin\alpha = \frac{y}{r} = \frac{15}{17} \qquad \cos\alpha = \frac{x}{r} = \frac{-8}{17} \qquad \tan\alpha = \frac{y}{x} = \frac{15}{-8}$$

$$\csc\alpha = \frac{r}{y} = \frac{17}{15} \qquad \sec\alpha = \frac{r}{x} = \frac{17}{-8} \qquad \cot\alpha = \frac{x}{y} = \frac{-8}{15}$$

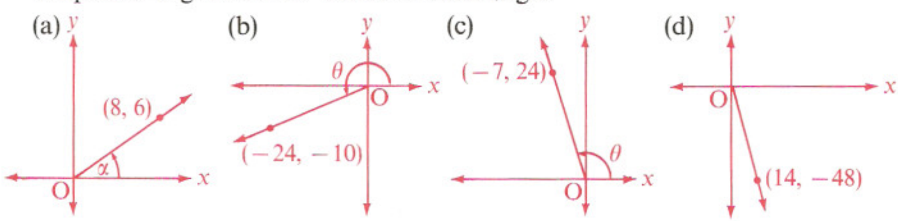
4.1 Exercise

A You may express answers for trigonometric values in fractional or radical form.

1 For each angle, a point on the terminal arm is shown. Calculate the primary trigonometric values.



2 A point on the terminal arm for each angle is shown. Calculate the reciprocal trigonometric values of each angle.



3 The point P(2, 7) lies on the terminal arm of θ. For θ, find the
(a) sine (b) cosine (c) tangent

4 The point Q(-9, 11) lies on the terminal arm of α. For α, calculate its
(a) cosecant (b) secant (c) cotangent

B 5 (a) Angle θ is a second quadrant angle, and $\cos \theta = -\frac{3}{4}$. Sketch a diagra
(b) Find the other primary trigonometric values of θ.

6 (a) Angle θ is in the third quadrant and $\tan \theta = \frac{3}{4}$. Find the reciprocal trigonometric values.
(b) Angle α is in the fourth quadrant and $\cos \alpha = \frac{8}{15}$. Find the other trigonometric values.

7 (a) θ is a first quadrant angle. If $\cos \theta = \frac{1}{\sqrt{5}}$, find $\sin \theta$ and $\sec \theta$.
(b) β is a second quadrant angle. If $\tan \beta = \frac{7}{-\sqrt{65}}$, find $\cos \beta$ and $\csc \beta$.

8 Draw a sketch of each angle in standard position. Calculate the other trigonometric values of each angle.
(a) $\sec \theta = -\frac{13}{12}$, θ in third quadrant (b) $\sin \alpha = \frac{7}{25}$, α in first quadrant (c) $\csc \theta = -\frac{17}{8}$, θ in fourth quadra

9 P(x, y) is a point on the terminal arm of α. OP $= r$.
(a) Find the possible values of k in each of the following.
(b) Find the primary trigonometric values for each value of k.
(i) P(3, k), $r = 5$ (ii) P(k, 8), $r = 10$ (iii) P(3, k), $r = \sqrt{13}$

10 As in algebra, $\sin^2 \alpha$ means $(\sin \alpha)^2$. α is a third quadrant angle and $\tan \alpha = \frac{5}{12}$. Find a value for $\sin^2 \alpha + \cos^2 \alpha$.

11 β is a second quadrant angle and $\csc \beta = \frac{17}{15}$. Find a value for $2 \sin \beta + 3 \cos \beta$.

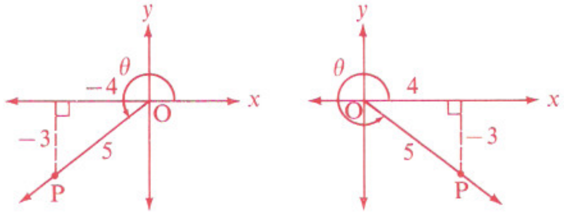
C 12 For any angle θ, show why

(a) $\sin \theta = \frac{1}{\csc \theta}$	(b) $\cos \theta = \frac{1}{\sec \theta}$	(c) $\tan \theta = \frac{1}{\cot \theta}$
(d) $\csc \theta = \frac{1}{\sin \theta}$	(e) $\sec \theta = \frac{1}{\cos \theta}$	(f) $\cot \theta = \frac{1}{\tan \theta}$

However, if you do not know what quadrant an angle is in, you can sketch a diagram to show the possibilities, as shown in Example 2.

Example 2 Find $\cos\theta$ if θ is positive and $\sin\theta = -\frac{3}{5}$.

Solution If θ is a positive angle and $\sin\theta = -\frac{3}{5}$, then θ may be an angle in the third or fourth quadrant.



Third Quadrant
$x = -4, y = -3, r = 5$

$$\cos\theta = -\frac{4}{5}$$

Fourth Quadrant
$x = 4, y = -3, r =$

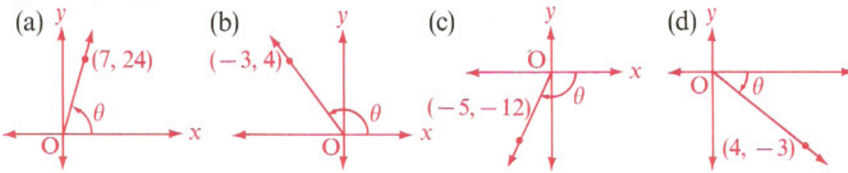
$$\cos\theta = \frac{4}{5}$$

4.2 Exercise

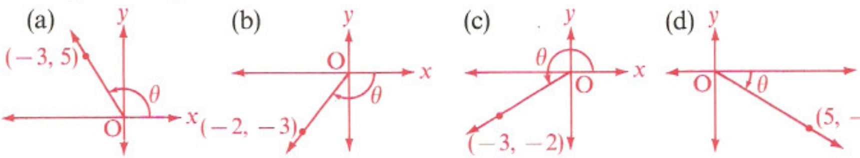
A Throughout the exercise you may leave answers in fractional or radical for

1 Draw a sketch of each angle in standard position.
(a) 120° (b) −60° (c) 135° (d) −225° (e) 210°

2 For each angle θ, a point on the terminal arm is given. Calculate the primary trigonometric ratios for θ.



3 For each angle, a point on the terminal arm is shown. Calculate the reciprocal trigonometric ratios.



4 α is an angle in the third quadrant and $\cos\alpha = \frac{-\sqrt{3}}{2}$.
(a) Write the co-ordinates of a point on the terminal arm.
(b) Find $\sin\alpha$ and $\tan\alpha$.

5 θ is an angle in the second quadrant and $\csc \theta = \frac{17}{15}$.

(a) Write the co-ordinates of a point on the terminal arm.

(b) Find $\cos \theta$, $\sec \theta$, and $\cot \theta$.

B 6 Given that $\cos \theta = -\frac{7}{25}$,

(a) In which possible quadrants can the terminal arm be placed?

(b) Draw a diagram to show each case in (a).

(c) Calculate the trigonometric values of $\sin \theta$.

7 β is an angle in standard position and $\sin \beta = \frac{4}{5}$.

(a) In which quadrants is it possible for the terminal arm to lie?

(b) Draw a diagram to show each case in (a).

(c) Calculate values for $\cos \beta$ and $\tan \beta$.

8 You know that $\cot \alpha = -\frac{24}{7}$.

(a) In which quadrants is it possible for the terminal arm to lie?

(b) Draw a diagram to show each case in (a).

(c) Calculate values for $\sin \alpha$ and $\cos \alpha$.

9 Examine the possibilities of each of the following.

(a) If $\sin \theta = \frac{-8}{17}$, find two values of $\cos \theta$.

(b) Given that $\cot \alpha = -\frac{12}{5}$. Find two values of $\sin \alpha$.

(c) For $\sec \beta = -\frac{25}{7}$, find $\tan \beta$.

(d) θ is in standard position. If $\cos \theta = \frac{-\sqrt{3}}{2}$, find $\cot \theta$.

10 If $\cos \theta = -\frac{7}{25}$, then calculate values for

(a) $\sin \theta$ (b) $(\sin \theta)(\cos \theta)$ (c) $(\cot \theta)(\tan \theta)$

C 11 If $\sin \theta = -\frac{3}{5}$, prove that $\sin^2 \theta + \cos^2 \theta = 1$

12 (a) $\cot \theta = -\frac{15}{8}$. Prove $\frac{\sin \theta}{\cos \theta} = \tan \theta$. (b) $\cos \theta = -\frac{\sqrt{3}}{2}$. Prove $\frac{\sin \theta}{\cos \theta} = \tan \theta$.

(c) What probable conclusion seems true based on your results in (a) and (b)?