Exploring the Behaviour of Functions Near the Asymptotes

4.7

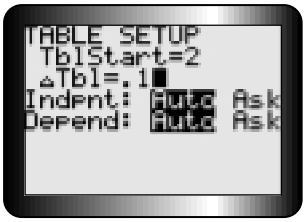
EXPLORATION

All of the reciprocal functions in the previous section had vertical and horizontal asymptotes. You can better understand the behaviour of the function near the asymptotes by looking at a table in a graphing calculator.

Examining the Function's Behaviour Near a Vertical Asymptote

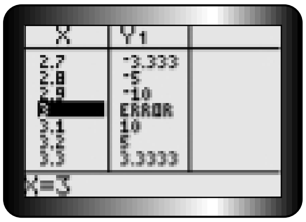
1. Adjust the window. Press [WINDOW] and enter these values: Xmin=−9.4, Xmax=9.4, Ymin=−5, and Ymax=5. Enter the reciprocal function $f(x) = \frac{1}{x-3}$. Press [GRAPH]. The graph has a vertical asymptote at $x = 3$, and the x-axis is a horizontal asymptote.

2. (a) You can examine what happens as the graph approaches the vertical asymptote by looking at the table as x approaches the value 3. Open the TABLE SETUP screen by pressing [2nd] [WINDOW] and enter the values shown here.



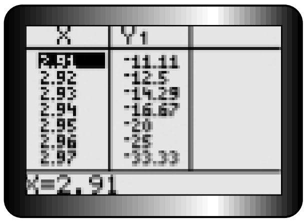
step 2a

(b) Display the table by pressing [2nd] [GRAPH]. Notice that the x-values begin at 2.0 and increase in increments of 0.1. Scroll down, using [▼], and observe the corresponding y-values. When you get to 3, there is no y-value, the word ERROR appears instead.



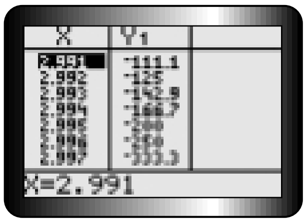
step 2b

3. (a) You can get closer to the asymptote by adjusting the values in the TABLE SETUP screen. Press [2nd] [WINDOW]. Change TblStart to 2.91 and ΔTbl to .01. ΔTbl defines the increment for the independent variable. Display the table again and scroll down. How do the y-values change as x approaches 3?



step 3a

(b) Get even closer by repeating step 3 and changing TblStart to 2.991 and ΔTbl to .001. As you scroll down the table, notice that the y-values move quickly away from 0. In the table, what is the smallest y-value at the point closest to the asymptote?

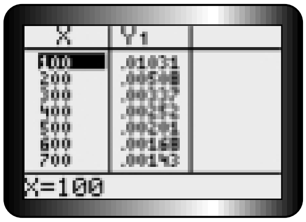


step 3b

4. If you continue scrolling past ERROR, then you are looking at points on the other side of the asymptote. Notice that the sign of y has changed. The y-value is just as far from zero, but is on the other side of zero.

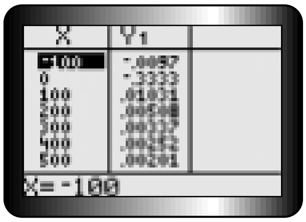
Examining the Function's Behaviour Near a Horizontal Asymptote

5. The graph approaches a horizontal asymptote when x is far from zero. Examine this part of the table. Press [2nd][WINDOW] and set TblStart to 100 and ΔTbl to 100. Press [2nd] [GRAPH]. Scroll down the table and look at the y-values.



step 5

6. Repeat step 5. Set TblStart to -100 and leave ΔTbl at 100. Scroll up to see what happens to y as x moves farther and farther away from the origin. As x moves farther away from the origin, in both directions along the x-axis, the y-values approach 0. The equation of the horizontal asymptote is $y = 0$.



step 6

7. For each function,
   i. enter the function into the graphing calculator
   ii. use a table to examine the behaviour of the graph near the vertical asymptote(s)
   iii. sketch the graph near the vertical asymptote(s)
   iv. use the table to examine the behaviour of the graph where x is far from 0. Write the equation of the horizontal asymptote. Remember to check positive and negative x-values.

(a) $f(x) = \dfrac{2x}{x - 4}$

(b) $g(x) = \dfrac{3x}{2 - x}$

(c) $h(x) = \dfrac{x^2 + 3}{x^2 - 5}$

(d) $h(x) = \dfrac{x^2 - 4}{x^2 + 5x + 6}$

(e) $g(x) = \dfrac{1}{x^2 - 1}$

(f) $f(x) = \dfrac{2x}{x^2 + 4x + 3}$

(g) $b(x) = \dfrac{x + 1}{x - 2}$

(h) $m(x) = \dfrac{x}{x^2 - x - 6}$

EXPLORATION