EXAMPLE 2: Solve $[(5x)^2]^2 = (2x + 3)^4$.

First, rewrite the equation so that exponents are the same.

$$((5x)^2)^2 = (2x + 3)^4$$
$$(5x)^4 = (2x + 3)^4$$

Since exponents are the same and even, the bases are either equal or equal with opposite signs.

$$5x = \pm(2x + 3)$$

$$5x = 2x + 3 \qquad 5x = -(2x + 3)$$
$$3x = 3 \qquad 7x = -3$$

There are two solutions.

$$x = 1 \qquad x = -\tfrac{3}{7}$$

Sometimes, it may be necessary to use a combination of different properties of exponents when solving exponential equations.

EXAMPLE 3: Solve $(x - 5)^{\frac{2}{3}} = \left(\frac{1}{27}\right)^{-\frac{1}{9}}$.

You can eliminate fractions in exponents by cubing both sides of the equation.

$$(x - 5)^{\frac{2}{3}} = \left(\frac{1}{27}\right)^{-\frac{1}{9}}$$
$$\left[(x - 5)^{\frac{2}{3}}\right]^3 = \left[\left(\frac{1}{27}\right)^{-\frac{1}{9}}\right]^3$$

Simplify exponents.

$$(x - 5)^2 = \left(\frac{1}{27}\right)^{-\frac{1}{3}} \qquad \left(\frac{1}{27}\right) = 27^{\frac{1}{3}}$$

There are two solutions.

$$(x - 5)^2 = 3$$
$$x - 5 = \sqrt{3} \text{ or } x - 5 = -\sqrt{3}$$
$$x = 5 + \sqrt{3} \text{ or } x = 5 - \sqrt{3}$$

EXERCISE 2-3

A **1.** Solve for x.

a. $2^x = 2$ **b.** $3^x = 27$ **c.** $2^x = 16$ **d.** $2^x = \frac{1}{4}$

e. $3^x = \frac{1}{9}$ **f.** $2^x = \frac{1}{16}$ **g.** $5^x = \frac{1}{125}$ **h.** $5^x = \sqrt{125}$

i. $7^x = 7\sqrt{7}$ **j.** $2^x = 8^3$ **k.** $5^{x+4} = 25$ **l.** $3^{2+x} = 81$

2. Solve each equation. There may be more than one solution for each equation.

a. $(x + 2)^4 = 5^4$ **b.** $(y - 1)^3 = 4^3$ **c.** $(x - 3)^5 = 1$ **d.** $(y + 2)^2 = 1$

3. Rewrite each equation without fractional exponents by raising each side of the equation to the same power.

a. $x^{\frac{2}{3}} = 8$ **b.** $y^{-\frac{3}{2}} = 8$ **c.** $z^{\frac{3}{4}} = 27$

d. $x^{-\frac{3}{4}} = 8$ **e.** $y^{\frac{5}{3}} = 32$ **f.** $z^{-\frac{3}{2}} = 216$

B 4. Solve for x.

a. $3^x = \sqrt{27}$ b. $9^{1+x} = 3$ c. $9^{x-7} = 27$ d. $\left(\frac{1}{4}\right)^{x+4} = 8$

e. $9^{1-x} = 27$ f. $8^{2x-1} = 2$ g. $64^{x-2} = 16^{4x}$ h. $3^{5-x} = \frac{1}{3}$

i. $2^{3x+4} = 0.25$ j. $49^{x-1} = 7\sqrt{7}$ k. $121^{x-2} = 11\sqrt{11}$ l. $10^{x-4} = 100^{4-x}$

5. Solve each equation.

a. $(2x + 1)^4 = (3x - 5)^4$ b. $(2x - 1)^5 = (x + 1)^5$ c. $(5x + 2)^4 = [(4x)^2]^2$

d. $\sqrt{6x - 3} = (2x - 5)^{\frac{1}{2}}$ e. $\sqrt[3]{7x - 3} = (5x + 7)^{\frac{1}{3}}$ f. $(9x^2)^2 = (2x + 9)^4$

6. A radioactive substance decays according to the formula

$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where A_0 is the initial mass of the substance,

A is the mass remaining after time t, and
h is the half-life of the substance given in the same units as that of time. (Half-life is the time it takes for a substance to lose half of its radioactivity.)

a. The half-life of a radioactive substance is 4.0 h. How much of a 240 g sample of the substance remains after 12 h?
b. Cobalt-58 has a half-life of 9.0 h. How many grams of a 256 g cobalt sample remain after 31.5 h?
c. Iodine-131 has a half-life of 8 d. How much of a 200.0 g sample of iodine remains after 20 d?
d. The half-life of a radioactive substance is 12 d. How much of a 500 g sample of the radioactive substance remains after 40 h? after 1000 h?

7. A laboratory received 200 g of radioactive radon and 16 d later 12.5 g of the radioactive material remained. What is the half-life of the radon?

C 8. Solve for x. Find a common factor first.

a. $3^{x+1} + 3^x = 324$ b. $4^{x+1} + 4^x = 160$ c. $2^{x+2} + 2^x = 320$ d. $2^{x+2} - 2^x = 96$

9. Solve for x.

a. $\dfrac{3^{x^2}}{3^x} = 9^3$ b. $\dfrac{2^{x^2}}{(2^x)^2} = 8$ c. $\sqrt{\dfrac{9^{x+3}}{27^x}} = 81$

10. Solve.

a. $4^{2x} - 8(4)^x + 16 = 0$ b. $3^{2x} - 26(3)^x - 27 = 0$

c. $5^{2x} - 26(5)^x + 25 = 0$ d. $2^{2x-1} - 3(2^{x-1}) + 1 = 0$

e. $2^{\frac{2x}{3}+1} - 3(2^{\frac{x}{3}}) - 20 = 0$ f. $4^x - 9^x = 0$