EXERCISES 10.5

1. The table shows the temperature in an office block over a 36 hour period.

t (hr)	0	3	6	9	12	15	18	21	24	27	30	33	36
T °C	18.3	15.0	14.1	16.0	19.7	23.0	23.9	22.0	18.3	15.0	14.1	16.0	19.7

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.
(c) Use your rule to predict the temperature after 40 hours.

2. The table shows the light level (L) during an experiment on dye fading.

t (hr)	0	1	2	3	4	5	6	7	8	9	10
L	6.6	4.0	7.0	10.0	7.5	4.1	6.1	9.8	8.3	4.4	5.3

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.

3. The table shows the value in $s of an industrial share over a 20 month period.

Month	0	2	4	6	8	10	12	14	16	18	20
Value	7.0	11.5	10.8	5.6	2.1	4.3	9.7	11.9	8.4	3.2	2.5

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.

4. The table shows the population (in thousands) of a species of fish in a lake over a 22 year period.

Year	0	2	4	6	8	10	12	14	16	18	20	22
Pop	11.2	12.1	13.0	12.7	11.6	11.0	11.6	12.7	13.0	12.1	11.2	11.2

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.

5. The table shows the average weekly sales (in thousands of $s) of a small company over a 15 year period.

Time	0	1.5	3	4.5	6	7.5	9	10.5	12	13.5	15
Sales	3.5	4.4	7.7	8.4	5.3	3.3	5.5	8.5	7.6	4.3	3.6

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.

6. The table shows the average annual rice production, P, (in thousands of tonnes) of a province over a 10 year period.

t (yr)	0	1	2	3	4	5	6	7	8	9	10
P	11.0	11.6	10.7	10.5	11.5	11.3	10.4	11.0	11.6	10.7	10.5

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.

7. The table shows the depth of water (D metres) over a 5 second period as waves pass the end of a pier.

t (sec)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
D	11.3	10.8	10.3	10.2	10.4	10.9	11.4	11.7	11.8	11.5	11.0

(a) Estimate the amplitude, period, horizontal and vertical translations.
(b) Find a rule that models the data.

8. The population (in thousands) of a species of butterfly in a nature sanctuary is modelled by the function:

$$P = 3 + 2\sin\left(\frac{3\pi t}{8}\right), 0 \le t \le 12$$

where t is the time in weeks after scientists first started making population estimates.

(a) What is the initial population?
(b) What are the largest and smallest populations?
(c) When does the population first reach 4 thousand butterflies?

9. A water wave passes a fixed point. As the wave passes, the depth of the water (D metres) at time t seconds is modelled by the function:

$$D = 7 + \frac{1}{2}\cos\left(\frac{2\pi t}{5}\right), t > 0$$

(a) What are the greatest and smallest depths?
(b) Find the first two times at which the depth is 6.8 metres.

10. The weekly sales (S) (in hundreds of cans) of a soft drink outlet is modelled by the function:

$$S = 13 + 5.5\cos\left(\frac{\pi t}{6} - 3\right), t > 0$$

t is the time in months with $t = 0$ corresponding to January 1st 1990,

(a) Find the minimum and maximum sales during 1990.
(b) Find the value of t for which the sales first exceed 1500 ($S = 15$).
(c) During which months do the weekly sales exceed 1500 cans?

11. The rabbit population, $R(t)$ thousands, in a northern region of South Australia is modelled by the equation $R(t) = 12 + 3\cos\left(\frac{\pi}{6}t\right), 0 \le t \le 24$, where t is measured in months after the first of January.

(a) What is the largest rabbit population predicted by this model?
(b) How long is it between the times when the population reaches consecutive peaks?
(c) Sketch the graph of $R(t)$ for $0 \le t \le 24$.
(d) Find the longest time span for which $R(t) \ge 13.5$.
(e) Give a possible explanation for the behaviour of this model.

12. A hill has its cross-section modelled by the function,

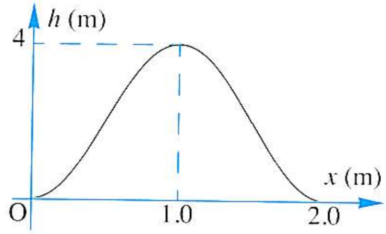
$$h : [0, 2] \mapsto \mathbb{R},\ h(x) = a + b\cos(kx),$$

where $h(x)$ measures the height of the hill relative to the horizontal distance x m from O.

(a) Determine the values of
  i. k.
  ii. b.
  iii. a.

(b) How far, horizontally from O, would an ant climbing this hill from O be, when it first reaches a height of 1 metre?

(c) How much further, horizontally, will the ant have travelled when it reaches the same height of 1 metre once over the hill and on its way down?



13. A nursery has been infested by two insect pests: the Fruitfly and the Greatfly. These insects appear at about the same time that a particular plant starts to flower. The number of Fruitfly (in thousands), t weeks after flowering has started is modelled by the function

$$F(t) = 6 + 2\sin(\pi t),\ 0 \le t \le 4.$$

Whereas the number of Greatfly (in thousands), t weeks after flowering has started is modelled by the function

$$G(t) = 0.25t^2 + 4,\ 0 \le t \le 4$$

(a) Copy and complete the following table of values, giving your answers correct to the nearest hundred.

t	0	0.5	1	1.5	2	2.5	3	3.5	4
$F(t)$									
$G(t)$									

(b) On the same set of axes **draw** the graphs of
  i. $F(t) = 6 + 2\sin(\pi t),\ 0 \le t \le 4$.
  ii. $G(t) = 0.25t^2 + 4,\ 0 \le t \le 4$.

(c) On how many occassions will there be equal numbers of each insect?

(d) For what percentage of the time will there be more Greatflies than Fruitflies?

14. The depth, $d(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given

$$d(t) = 12 + 3\sin\left(\frac{\pi}{6}t\right),\ 0 \le t \le 24$$

(a) Sketch the graph of $d(t)$ for $0 \le t \le 24$.

(b) For what values of t will
  i. $d(t) = 10.5,\ 0 \le t \le 24$.
  ii. $d(t) \ge 10.5,\ 0 \le t \le 24$.

Boats requiring a minimum depth of b metres are only permitted to enter the harbour when the depth of water at the entrance of the harbour is at least b metres for a continuous period of one hour.

(c) Find the largest value of b, correct to two decimal place, which satisfies this condition.