EXERCISES 10.4

1. If $0 \leq x \leq 2\pi$, find:

(a) $\sin x = \dfrac{1}{\sqrt{2}}$ 　　(b) $\sin x = -\dfrac{1}{2}$ 　　(c) $\sin x = \dfrac{\sqrt{3}}{2}$

(d) $\sin 3x = \dfrac{1}{2}$ 　　(e) $\sin\left(\dfrac{x}{2}\right) = \dfrac{1}{2}$ 　　(f) $\sin(\pi x) = -\dfrac{\sqrt{2}}{2}$

2. If $0 \leq x \leq 2\pi$, find:

(a) $\cos x = \dfrac{1}{\sqrt{2}}$ 　　(b) $\cos x = -\dfrac{1}{2}$ 　　(c) $\cos x = \dfrac{\sqrt{3}}{2}$

(d) $\cos\left(\dfrac{x}{3}\right) = \dfrac{1}{2}$ 　　(e) $\cos(2x) = \dfrac{1}{2}$ 　　(f) $\cos\left(\dfrac{\pi}{2}x\right) = -\dfrac{\sqrt{2}}{2}$

3. If $0 \leq x \leq 2\pi$, find:

(a) $\tan x = \dfrac{1}{\sqrt{3}}$ 　　(b) $\tan x = -1$ 　　(c) $\tan x = \sqrt{3}$

(d) $\tan\left(\dfrac{x}{4}\right) = 2$ 　　(e) $\tan(2x) = -\sqrt{3}$ 　　(f) $\tan\left(\dfrac{\pi}{4}x\right) = -1$

4. If $0 \leq x \leq 2\pi$ or $0 \leq x \leq 360$, find:

(a) $\sin(x° + 60°) = \dfrac{1}{2}$ 　　(b) $\cos(x° - 30°) = -\dfrac{\sqrt{3}}{2}$

(c) $\tan(x° + 45°) = -1$ 　　(d) $\sin(x° - 20°) = \dfrac{1}{\sqrt{2}}$

(e) $\cos\left(2x - \dfrac{\pi}{2}\right) = \dfrac{1}{2}$ 　　(f) $\tan\left(\dfrac{\pi}{4} - x\right) = 1$

5. If $0 \leq x \leq 2\pi$ or $0 \leq x \leq 360$, find:

(a) $\cos x° = \dfrac{1}{2}$ 　　(b) $2\sin x + \sqrt{3} = 0$ 　　(c) $\sqrt{3}\tan x = 1$

(d) $5\sin x° = 2$ 　　(e) $4\sin^2 x - 3 = 0$ 　　(f) $\dfrac{1}{\sqrt{3}}\tan x + 1 = 0$

(g) $2\sin\left(x + \dfrac{\pi}{3}\right) = -1$ 　(h) $5\cos(x + 2) - 3 = 0$ 　　(i) $\tan\left(x - \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{3}}$

(j) $2\cos 2x + 1 = 0$ 　　(k) $\tan 2x - \sqrt{3} = 0$ 　　(l) $2\sin x° = 5\cos x°$

6. Solve the following equations for the intervals indicated, giving exact answers:

(a) $\sin\theta\cos\theta = \dfrac{1}{2}, -\pi \leq \theta \leq \pi$ 　　(b) $\cos^2\theta - \sin^2\theta = -\dfrac{1}{2}, -\dfrac{\pi}{2} \leq \theta \leq \dfrac{\pi}{2}$

(c) $\tan A = \dfrac{1 - \tan^2 A}{2}, -\pi \le A \le \pi$ (d) $\dfrac{\sin\theta}{1 + \cos\theta} = -1, -\pi \le \theta \le \pi$

(e) $\cos^2 x = 2\cos x, -\pi \le x \le \pi$ (f) $\sec 2x = \sqrt{2}, 0 \le x \le 2\pi$

(g) $2\sin^2 x - 3\cos x = 2, 0 \le x \le 2\pi$ (h) $\sin 2x = 3\cos x, 0 \le x \le 2\pi$

7. Find (a) $3\tan^2 x + \tan x = 2, 0 \le x \le 2\pi$.

   (b) $\tan^3 x + \tan^2 x = 3\tan x + 3, 0 \le x \le 2\pi$.

8. If $0 \le x \le 2\pi$ find:

   (a) $\sin^2 2x - \dfrac{1}{4} = 0$ (b) $\tan^2\left(\dfrac{x}{2}\right) - 3 = 0$ (c) $\cos^2(\pi x) = 1$

9. (a) Sketch the graph of $f(x) = \sin x, 0 \le x \le 4\pi$.

   (b) Hence, find i. $\left\{x \,\middle|\, \sin x > \dfrac{1}{2}\right\} \cup \{x \,|\, 0 < x < 4\pi\}$.

   ii. $\{x \,|\, \sqrt{3}\sin x < -1\} \cup \{x \,|\, 0 < x < 4\pi\}$.

10. (a) i. On the same set of axes sketch the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ for $0 \le x \le 2\pi$.

   ii. Find $\{x \,|\, \sin x < \cos x, 0 \le x \le 2\pi\}$.

   (b) i. On the same set of axes sketch the graphs of $f(x) = \sin 2x$ and $g(x) = \cos x$ for $0 \le x \le 2\pi$.

   ii. Find $\{x \,|\, \sin 2x < \cos x, 0 \le x \le 2\pi\}$.

11. (a) Solve $\left\{x^\circ : 3\sin x^\circ - \dfrac{1}{\sin x^\circ} = 2, 0^\circ \le x \le 360^\circ\right\}$

   (b) Hence, find $\left\{x^\circ : 3\sin x^\circ < \dfrac{1}{\sin x^\circ} + 2, 0^\circ \le x \le 360^\circ\right\}$.