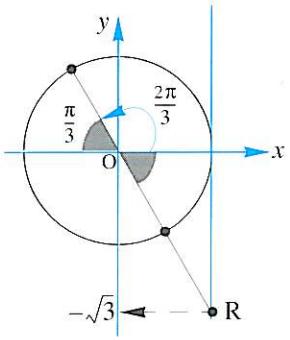
(b) This time we are searching for those values of θ for which $\tan\theta = -\sqrt{3}$.
From the unit circle, we extend the ray OP so that it cuts the tangent line at R.



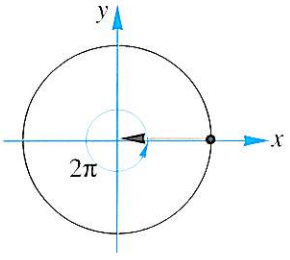
Using the exact values, we have $\tan\left(\pi - \frac{\pi}{3}\right) = \tan\frac{2\pi}{3} = -\sqrt{3}$

(as our first value).

And, by symmetry, we also have that $\tan\left(2\pi - \frac{\pi}{3}\right) = -\sqrt{3}$.

Therefore, $\theta = \frac{2\pi}{3}$ or $\theta = \frac{5\pi}{3}$.

(c) $\cos\theta = 1$, therefore, $\theta = 0$ or $\theta = 2\pi$



EXAMPLE 10.6

Simplify

(a) $\dfrac{\sin(\pi+\theta)}{\cos(2\pi-\theta)}$ (b) $\dfrac{\sin\left(\frac{\pi}{2}+\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)}{\cos(\pi+\theta)}$, where $0 < \theta < \frac{\pi}{2}$.

Solution

(a) $$\frac{\sin(\pi+\theta)}{\cos(2\pi-\theta)} = \frac{-\sin\theta}{\cos\theta}$$
$$= -\tan\theta$$

(b) $$\frac{\sin\left(\frac{\pi}{2}+\theta\right)\cos\left(\frac{\pi}{2}-\theta\right)}{\cos(\pi+\theta)} = \frac{\cos\theta\sin\theta}{-\cos\theta}$$
$$= -\sin\theta$$

EXERCISES 10.1

1. Convert the following angles to degrees.

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{5}$ (c) $\frac{12\pi}{10}$ (d) $\frac{5\pi}{18}$

2. Convert the following angles to radians.

(a) 180° (b) 270° (c) 140° (d) 320°

3. Find the exact value of

(a) $\sin 120°$ (b) $\cos 120°$ (c) $\tan 120°$ (d) $\sin 210°$

(e) $\cos 210°$ (f) $\tan 210°$ (g) $\sin 225°$ (h) $\cos 225°$

(i) $\tan 225°$ (j) $\sin 315°$ (k) $\cos 315°$ (l) $\tan 315°$

(m) $\sin 360°$ (n) $\cos 360°$ (o) $\tan 360°$

4. Find the exact value of

(a) $\sin \pi$ (b) $\cos \pi$ (c) $\tan \pi$ (d) $\sin \frac{3\pi}{4}$

(e) $\cos \frac{3\pi}{4}$ (f) $\tan \frac{3\pi}{4}$ (g) $\sin \frac{7\pi}{6}$ (h) $\cos \frac{7\pi}{6}$

(i) $\tan \frac{7\pi}{6}$ (j) $\sin \frac{5\pi}{3}$ (k) $\cos \frac{5\pi}{3}$ (l) $\tan \frac{5\pi}{3}$

(m) $\sin \frac{7\pi}{4}$ (n) $\cos \frac{7\pi}{4}$ (o) $\tan \frac{7\pi}{4}$

5. Find the exact value of

(a) $\sin(-210°)$ (b) $\cos(-30°)$ (c) $\tan(-135°)$ (d) $\cos(-420°)$

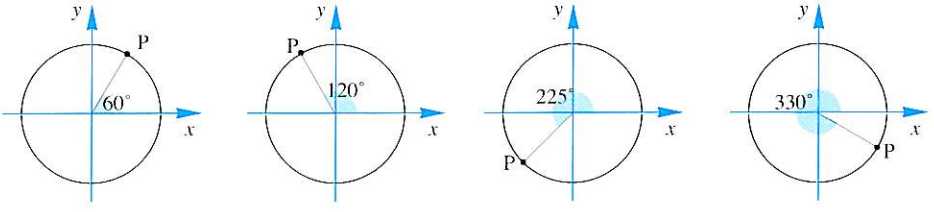
6. Find the exact value of

(a) $\sin\left(-\frac{\pi}{6}\right)$ (b) $\cos\left(-\frac{3\pi}{4}\right)$ (c) $\tan\left(-\frac{2\pi}{3}\right)$ (d) $\sin\left(-\frac{7\pi}{6}\right)$

(e) $\cos\left(-\frac{7\pi}{6}\right)$ (f) $\tan\left(-\frac{11\pi}{6}\right)$ (g) $\sin\left(-\frac{7\pi}{3}\right)$

7. Find the coordinates of the point P on the following unit circles.

(a) (b) (c) (d)



8. Find the exact value of

(a) $\sin \frac{11\pi}{6} \cos \frac{5\pi}{6} - \sin \frac{5\pi}{6} \cos \frac{11\pi}{6}$ (b) $2 \sin \frac{\pi}{6} \cos \frac{\pi}{6}$

(c) $\dfrac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}}$ (d) $\cos \frac{\pi}{4} \cos \frac{\pi}{3} + \sin \frac{\pi}{4} \sin \frac{\pi}{3}$

9. Show that the following relationships are true

(a) $\sin 2\theta = 2\sin\theta\cos\theta$, where $\theta = \frac{\pi}{3}$

(b) $\cos 2\theta = 2\cos^2\theta - 1$, where $\theta = \frac{\pi}{6}$.

(c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, where $\theta = \frac{2\pi}{3}$.

(d) $\sin(\theta - \phi) = \sin\theta\cos\phi - \sin\phi\cos\theta$, where $\theta = \frac{2\pi}{3}$ and $\phi = -\frac{\pi}{3}$.

10. Given that $\sin\theta = \frac{2}{3}$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\sin(\pi + \theta)$ (b) $\sin(2\pi - \theta)$ (c) $\cos\left(\frac{\pi}{2} + \theta\right)$

11. Given that $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\cos(\pi - \theta)$ (b) $\sin\left(\frac{\pi}{2} - \theta\right)$

12. Given that $\tan\theta = k$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\tan(\pi + \theta)$ (b) $\tan\left(\frac{\pi}{2} + \theta\right)$ (c) $\tan(-\theta)$

13. Given that $\sin\theta = \frac{2}{3}$ and $0 < \theta < \frac{\pi}{2}$ find

(a) $\cos\theta$ (b) $\cos(\pi + \theta)$

14. Given that $\cos\theta = -\frac{4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$ find

(a) $\sin\theta$ (b) $\tan\theta$ (c) $\cos(\pi + \theta)$

15. Given that $\tan\theta = -\frac{4}{3}$ and $\frac{\pi}{2} < \theta < \pi$ find

(a) $\sin\theta$ (b) $\tan\left(\frac{\pi}{2} + \theta\right)$

16. Given that $\cos\theta = k$ and $\frac{3\pi}{2} < \theta < 2\pi$ find

(a) $\cos(\pi - \theta)$ (b) $\sin\theta$

17. Given that $\sin\theta = -k$ and $\pi < \theta < \frac{3\pi}{2}$ find

(a) $\cos\theta$ (b) $\tan\theta$