REVIEW SOLUTIONS

1. Graph of quadratic function.

Expression	+	–	0
a		✓	
c		✓	
$b^2 - 4ac$			✓
b	✓		

(A1) (C1)
(A1) (C1)
(A1) (C1)
(A1) (C1)

[4]

2. (a) $2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8$ (M1)

$= 2(x - 2)^2 - 3$ (A1)(A1)(A1)

=> $a = 2$, $p = 2$, $q = -3$ (C4)

(b) Minimum value of $2(x - 2)^2 = 0$ (or minimum value occurs when $x = 2$) (Ml)

⇒ Minimum value of $f(x) = -3$ (A1) (C2)

OR

Minimum value occurs at (2, –3) (M1)(A1) (C2)

[6]

3. $5x^2 + 2kx + 2 = 0$

Only one solution ⇒ $b^2 - 4ac = 0$ (M1)

$4k^2 - 4(5)(2) = 0$ (A1)

$k^2 = 10$

$k = \pm\sqrt{10}$ (A1)

But given $k > 0$, $k = \sqrt{10}$ (A1) (C4)

[4]

4. $(7 - x)(1 + x) = 0$ (M1)

⇒ $x = 7$ or $x = -1$

A (-1, 0), C(7, 0) (A1)(C1)(C1)

B: $x = \dfrac{7 + -1}{2} = 3$; (A1)

$y = (7 - 3)(l + 3) = 16$

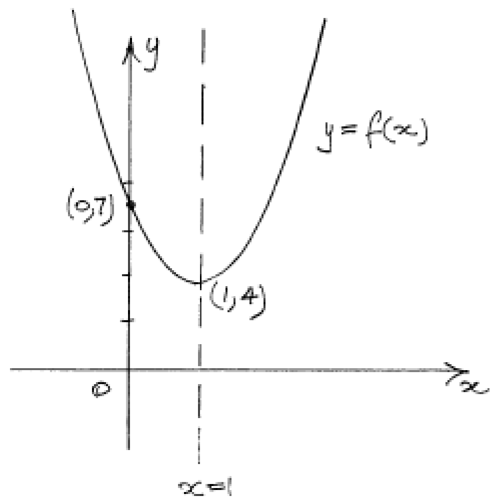
B(3, 16) (A1) (C2)

[4]

Q5. (a) $f(x) = 3x^2 - 6x + 7$
$= 3(x^2 - 2x + 1) + 4$
$= 3(x-1)^2 + 4$

(b) (i) $(1, 4)$
(ii) $x = 1$

(c) (i) $f(0) = 7 \quad \therefore (0, 7)$
(ii)



Q7. $g(x) = 2x^2 - 12x + 7$
$= 2(x^2 - 6x + 9) - 11$
$= 2(x-3)^2 - 11$

$g(x) = 0 \Rightarrow 2(x-3)^2 - 11 = 0$
$\Leftrightarrow (x-3)^2 = \frac{11}{2}$
$\Leftrightarrow x = 3 \pm \sqrt{\frac{11}{2}}$

$\therefore B \equiv \left(3 + \sqrt{\frac{11}{2}}, 0\right)$

$\Rightarrow a = 3, b = 11, c = 2$.

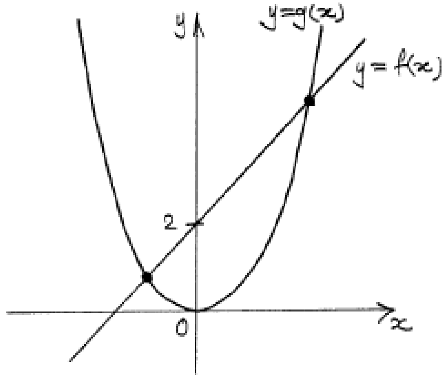
Q8. (a) (i) $g(2) = 2 + \frac{1}{2}$
$= 2.5$

(ii) $g(x) = 2 \Leftrightarrow x + \frac{1}{x} = 2$
$\Leftrightarrow x^2 + 1 = 2x$
$\Leftrightarrow x^2 - 2x + 1 = 0$
$\Leftrightarrow (x-1)^2 = 0$
$\Leftrightarrow x = 1$

(b) $(g \circ g)(x) = g(g(x))$
$= g(x) + \frac{1}{g(x)}$
$= \left(x + \frac{1}{x}\right) + \frac{1}{\left(x + \frac{1}{x}\right)}$
$= x + \frac{1}{x} + \frac{x}{x^2+1}$
$= \frac{x^2+1}{x} + \frac{x}{x^2+1}$
$= \frac{(x^2+1)^2 + x^2}{x(x^2+1)}$
$= \frac{x^4 + 2x^2 + 1 + x^2}{x(x^2+1)}$
$= \frac{x^4 + 3x^2 + 1}{x(x^2+1)}$

$\therefore f(x) = x^4 + 3x^2 + 1$

Q15. (a)



(b) (i) $f(x) = g(x) \Leftrightarrow 5x + 2 = 3x^2$

$\Leftrightarrow 3x^2 - 5x - 2 = 0$

$\Leftrightarrow (3x+1)(x-2) = 0$

$\Leftrightarrow x = -\frac{1}{3}$ or $x = 2$.

$\therefore$ Sol^n set $= \{2, -\frac{1}{3}\}$

Q19. $g(x) = ax - b$

$g(1) = 9 \Rightarrow 9 = a - b$ —— ①

$g(-3) = 1 \Rightarrow 1 = -3a - b$ —— ②

① − ②: $8 = 4a \Leftrightarrow a = 2$

Sub into ①: $9 = 2 - b \Leftrightarrow b = -7$.

Q24. (a) $f(x) = x^3$, $g(x) = x - 1$

$(f \circ g)(x) = f(g(x))$

$= (x-1)^3$

$\therefore (f \circ g)(1) = 0^3 = 0$

(b) $(g \circ f)(2) = g(f(2))$ $\qquad f(2) = 2^3 = 8$

$= g(8)$

$= 8 - 1$

$= 7$

Q26. $h(x) = 2x^2 - kx + k$

If $h(x) = 0$ has 2 distinct solutions

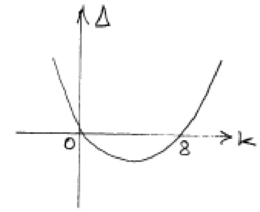
$\Rightarrow$ discriminant $(\Delta) > 0$

$\Delta = b^2 - 4ac$

$= (-k)^2 - 4(2)(k)$

$= k^2 - 8k$

$\therefore \Delta = k(k-8)$



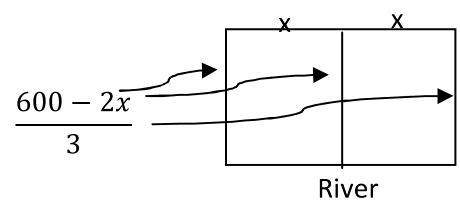
$\Delta > 0 \Rightarrow k(k-8) > 0$

$\Leftrightarrow k < 0$ or $k > 8$

$\therefore k \in]-\infty, 0[\cup]8, \infty[$

12. for a full solution, see
http://www.purplemath.com/modules/quadprob2.htm
The width of the pathway will be 1.5 meters.

13.



$$A = \left(\frac{600 - 2x}{3}\right) 2x$$

$$A = 400x - \frac{4}{3}x^2$$

Max area at vertex:

$$x = \frac{-400}{2(\frac{-4}{3})} \qquad A(150) = 30{,}000$$

$$= 150$$

The maximum area is 30,000 square ft.

14. for a full solution, see:
http://www.purplemath.com/modules/quadprob3.htm
I will minimize my costs if I produce 106 300 units a day.

15. (a) $0 = -4.9t^2 + 19.6t + 58.8$

$0 = t^2 - 4t - 12$

$0 = (t - 6)(t + 2)$

The balloon strikes the ground six seconds after launch.

(b) the initial height was 58.8 m

(c) height of 75 (from graph) $t \approx 1.17,\ 2.83$ (3 sig figs)

The balloon attains a height of 75 m at about 1.17 seconds and 2.83 seconds.

(d) Max height @ vertex.

$t = \frac{-2+6}{2}$, $t = 2$, $h(2) = 78.4$

The max height is 78.4 m