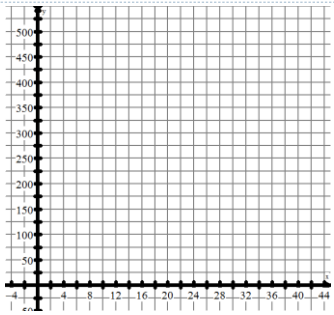
Lesson 9 – Special Functions

Math 2 Honors - Santowski

Fast Five

- When Mr S. was in high school, he had a part time job working on a neighbour's dairy farm. My wages (salary) was $10.00 per hour worked. If I worked more than 20 hours a week, I earned "time and a half" (my hourly pay was increased by 50%)
- Graph the relation showing my wages as a function of hours worked. Set up the graph showing the relation for up to 40 hours worked
- Write an equation for the relation.
- State the domain and range of the relation if I worked 30 hours.
- Is the relation a function?
- What would the inverse of the relation mean?

Fast Five



Fast Five

Lesson Objectives

- Define and graph piecewise functions, step functions, and absolute-value functions
- Use these special functions to review the following prior lesson objectives:
  - Evaluate functions
  - Analyze functions
  - Find compositions of functions
  - Find the inverse of a function

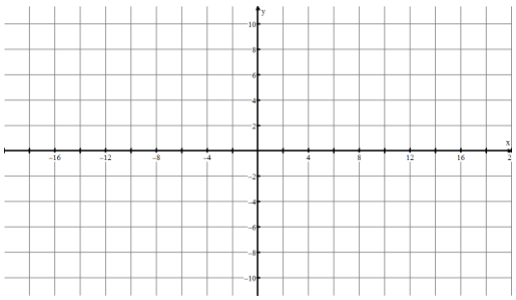
(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

$$f(x) = \begin{cases} 2x-1 & \text{if } x < 3 \\ -x & \text{if } x \geq 3 \end{cases}$$

- (a) state domain, range, and intercepts
- (b) NEW TERM: Is f(x) a continuous function?
- (c) Graph the inverse, $f^{-1}(x)$
- (d) Determine the domain and range of the inverse
- (e) is the inverse a function or not?

Grid to Use



(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

$$f(x)=\begin{cases}(x+3)^2+1 & x<-2\\ 2 & -2\le x<2\\ 5-x & x\ge 2\end{cases}$$

- (a) Graph y = f(x)
- (b) Evaluate f(-3); f(1); f(4)
- (c) Solve f(x) = 1 Solve f(x) = 3
- (d) State the domain and range of f(x)
- (e) NEW TERM: This function is not continuous. Why not and what would "continuous" mean wrt to functions

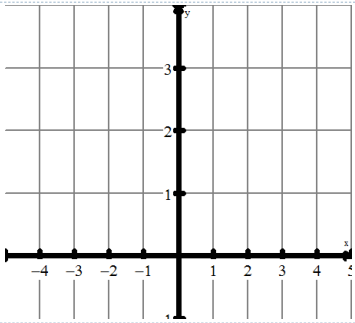
(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

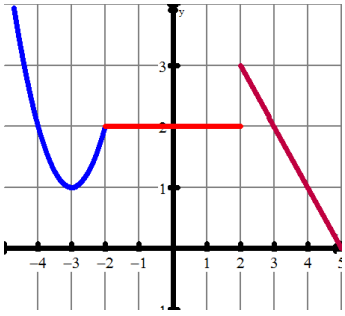
$$f(x)=\begin{cases}(x+3)^2+1 & x<-2\\ 2 & -2\le x<2\\ 5-x & x\ge 2\end{cases}$$

- (f) Graph the inverse relation
- (g) Determine the domain and range of the inverse
- (h) is the inverse a function or not?

(A) Piecewise Functions



(A) Piecewise Functions



(B) Step Functions

- One step function, the greatest integer function, is a function that takes an input and ROUNDS the input value DOWN to the nearest integral value:
- The notation is $f(x)=\lfloor x\rfloor$
- ex. of evaluations are:

$$f(3.2)=\lfloor 3.2\rfloor=$$
$$f(3.5)=\lfloor 3.5\rfloor=$$
$$f(3.9)=\lfloor 3.9\rfloor=$$
$$f(3.9999)=\lfloor 3.9999\rfloor=$$
$$f(-1.5)=\lfloor -1.5\rfloor=$$

(B) Greatest Integer Function

- Prepare a table of values and graph $f(x)=\lfloor x \rfloor$
- Now graph the following, given that $f(x)=\lfloor x \rfloor$
- (a) y = |f(x)|
- (b) y = $f^{-1}(x)$

(B) Step Functions

- Another step function, a ceiling function, is a function that takes an input and ROUNDS the input value UP to the nearest integral value (i.e. Phone companies who charge on a per minute basis)
- The notation is $f(x)=\lceil x \rceil$
- ex. of evaluations are:

$f(3.2)=\lceil 3.2 \rceil =$
$f(3.5)=\lceil 3.5 \rceil =$
$f(3.9)=\lceil 3.99 \rceil =$
$f(4.001)=\lceil 4.001 \rceil =$
$f(-1.5)=\lceil -1.5 \rceil =$

(C) Incorporating Function Concepts

- Determine the equation for, state domain, evaluate y(-2.2) and then graph y(x), given the following four functions that are used to define y(x):

$f(x)=|x|, \quad g(x)=\lfloor x \rfloor, \quad h(x)=2x-6, \quad t(x)=2-x^2$

$y(x)=f \circ t(x)$ $\quad y(x)=\dfrac{f(x)}{x}$

$y(x)=t \circ g(x)$ $\quad y(x)=h^{-1} \circ t^{-1}(x)$

$y(x)=h(x)+t(x)$ $\quad y(x)=\dfrac{f(x)}{h(x)}$

$y(x)=f \circ h(x)$ $\quad y(x)=\begin{cases} f(x) & x<3 \\ t(x) & x \geq 3 \end{cases}$

(C) Incorporating Function Concepts

- Are function operations associative??
- Use algebraic and graphic evidence to support your conclusions if

$f(x)=|x|, \quad h(x)=2x-6, \quad t(x)=2-x^2$

- (a) is addition? $(f(x)+t(x))+h(x)=?=f(x)+(t(x)+h(x))$
- (b) is multiplication? $(f(x)\times t(x))\times h(x)=?=f(x)\times(t(x)\times h(x))$
- (c) is composition? $(f(x)\circ t(x))\circ h(x)=?=f(x)\circ(t(x)\circ h(x))$

Homework

- p. 129 # 19,21,23,25, 26,27,29-37odds,53,55,57,66,68

(B) The Absolute Value Function

- Recall that the absolute value function was defined as a piecewise function as you just reviewed on the previous slide
- If f(x) = |x|, then evaluate the following:
  - (a) f(-1) + 5 f(-2) + 5
  - (b) f(-1 + 5) f(-2 + 5)
  - (c) -2f(-1) -2f(-2)
- If f(x) = |x|, then graph the following by using a table of values:
  - (a) y = f(x) + 5
  - (b) y = f(x + 5)
  - (c) y = -2f(x)

(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

$$f(x)=\begin{cases}-x & \text{if } x<0\\ x & \text{if } x\geq 0\end{cases}$$

- (a) state domain, range, and intercepts
- (b) NEW TERM: Is f(x) a continuous function?
- (c) Graph the inverse, $f^{-1}(x)$
- (d) Determine the domain and range of the inverse
- (e) is the inverse a function or not?