Lesson 8 - The Inverse Function

Math 2 Honors - Santowski

Fast Five – Skills Preview

- Define "inverse of a function"
- Graphically, explain what "inverse of a function" means.
- Find the inverse of:
- (a) $3x - 2y + 7 = 0$
- (b) $y = mx + b$
- (c) $y = \frac{1}{2}(x + 2)^2 - 5$

Lesson Objectives

- Find the inverse of a function from numeric/tabular, graphic or algebraic data
- Define a special class of functions called one-to-one functions.
- Use the horizontal line test to determine whether the inverse is a function
- Understand inverses as transformations
- Compose a function with its inverse to develop the identity function

The **BIG** Picture

- And we are studying this because ....?
- Are all functions invertible?
- Do function inverses "do the same thing" as our additive/multiplicative inverses?
- Why "invert" a function in the first place?

(A) Inverses - The Concept

- Let's back to our input ➔ output notion for functions.
- If functions are nothing more than input/output operators, then the concept of an inverse has us considering **how to go in reverse** ➔ going from the output back to the input

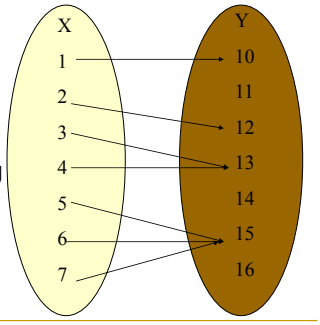
(A) Inverses - The Concept

- If the elements of the ordered pairs or mappings of a function are **reversed**, the resulting set of ordered pairs or mappings are referred to as the INVERSE.
- Another point worth noting: the domain of the original function now becomes the range of the inverse; likewise, the range of the original becomes the domain of the inverse.

(A) Inverses - The Concept

Determine:

(a) Domain of f(x)
(b) Range of f(x)
(c) Draw a mapping diagram for the inverse function
(a) Is the inverse a function?



(B) Notation of the Inverses

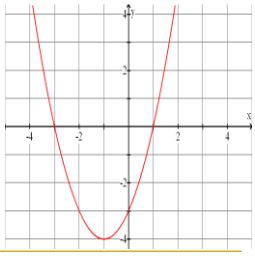
- The notation used for the inverses of functions is $f^{-1}(x)$.
- IMPORTANT NOTE: $f^{-1}(x)$ does not mean $(f(x))^{-1}$ or $1 / f(x)$.

(C) Examples

- Determine the equation for the inverse of the following functions. Draw both graphs and find the D and R of each.
- (1) y = 4x - 9
- (2) $y = 2x^2 + 4$
- (3) $y = 2 - \sqrt{(x+3)}$
- (4) $y = x^2 + 4x - 2$
- And some rational functions
- (5) $y = \frac{2}{3x-9}$
- (6) $y = \frac{2x-1}{3x-9}$
- (7) y = | x |

(C) Examples

- Consider a graph of the following data:
- 1. State domain and range of *f*
- 2. Evaluate f(-2), f(0),
- 3. Graph the inverse relation
- 4. Is the inverse a function?
- 5. HOW can we make it a function?
- 6. Evaluate $f^{-1}(1)$, $f^{-1}(-2)$
- 7. State the domain and range of $f^{-1}(x)$
- Here is the graph of f(x)



(C) Examples

- The cost of renting a car for a day is a flat rate of $40 plus $0.10/km
- 1. Write a function C(d) to represent the total cost of a one day rental. State restrictions on domain and range.
- 2. Find the equation of the inverse. What does the equation of the inverse represent?
- 3. Give an example of how the inverse could be used?

(C) Examples

- If an object is dropped from a height of 80 m, its height above the ground in meters is given by $h(t) = -5t^2 + 80$
- 1. Graph the function
- 2. Graph the inverse relation
- 3. Is the inverse a function
- 4. What does the inverse represent?
- 5. After what time is the object 35 m above the ground?
- 6. How long does the object take to fall?

(D) Composing with Inverses

- Let f(x) = 2x – 7.
- Determine the inverse of y = f(x)
- Graph both functions on a grid/graph
- Draw the line y = x. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $fof^{-1}(x)$ and $f^{-1}of(x)$. What do you notice?

(D) Composing with Inverses

- Now let $f(x) = x^2 + 2$.
- Determine the inverse of y = f(x)
- Graph both functions on a grid/graph
- Draw the line y = x. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $fof^{-1}(x)$ and $f^{-1}of(x)$. What do you notice?

(D) Composing with Inverses

- How can we use this observation?
- Determine the equation of the inverse of

$$f(x) = \frac{5}{x-3}$$

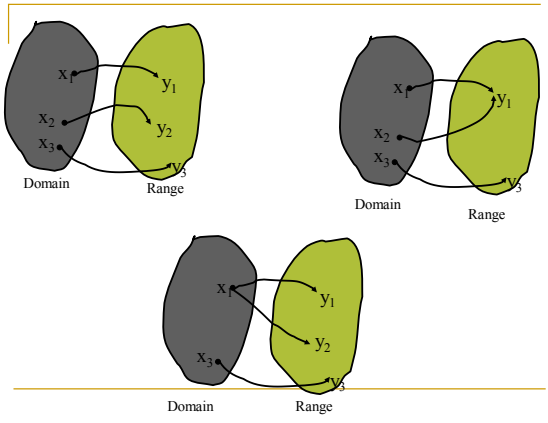
- Verify that your equation for the inverse IS correct (HINT: Composition)

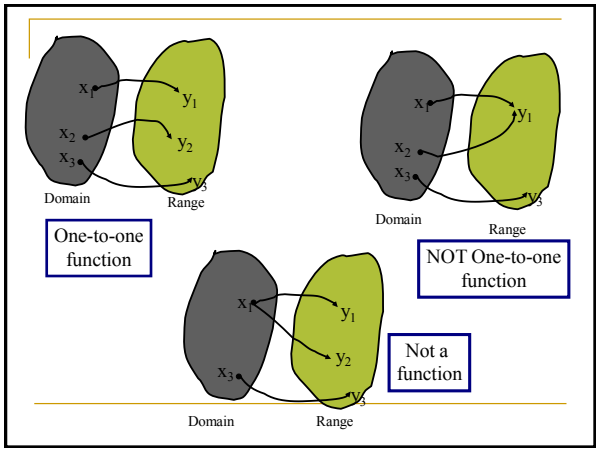
(E) Inverses as Transformations

- Place the ordered pairs on the Cartesian plane and we see the relationship between the original ordered pairs and the transformed ordered pairs of the inverse
- The relationship that exists is that the original points are reflected in the line y = x.

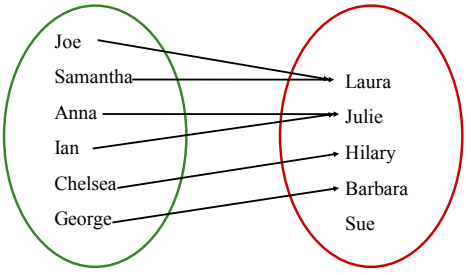
(F) One-to-One Functions

- A function *f* is **one-to-one** if for each *x* in the domain of *f* there is exactly one *y* in the range and no *y* in the range is the image of more than one *x* in the domain.
- A function is not one-to-one if two different elements in the domain correspond to the same element in the range.

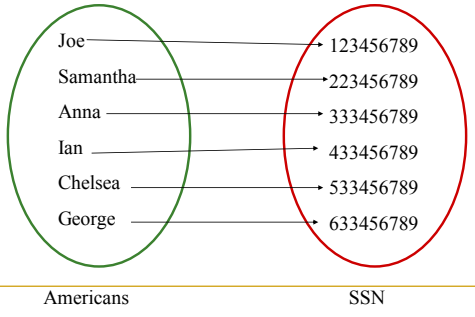




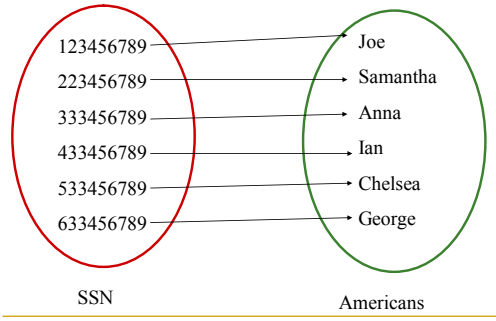
M: Mother Function is **NOT** one-one



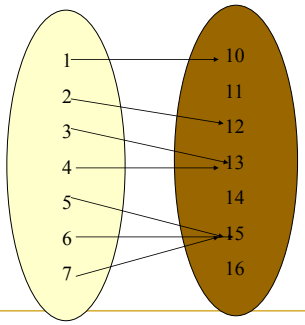
S: Social Security function **IS** one-one



S^{-1} The inverse of the social security function



Is the function f below one – one?

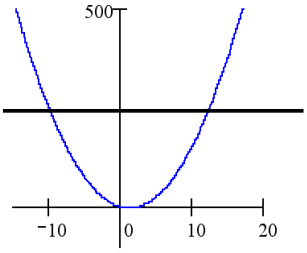


(F) One-to-One Functions - Examples

- Are the following functions one-to-one? Why or why not?

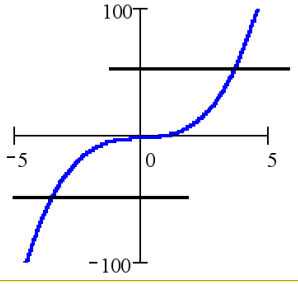
- (a) $f(x) = 2x^2 – 5x + 1$
- (b) $f(x) = x^3 + x – 2$
- (c) $h(x) = x(x-2)(x+1)$
- (c) $f(x) = \sin(x)$
- (d) $g(x) = 2^x$

Use the graph to determine whether the function $f(x) = 2x^2 - 5x + 1$ is one-to-one.



Not one-to-one.

Use the graph to determine whether the function $f(x) = x^3 + x - 2$ is one-to-one.



One-to-one.

Theorem Horizontal Line Test

- If horizontal lines intersect the graph of a function *f* in at most one point, then *f* is one-to-one.
- WHY does the Horizontal Line Test work?

(G) Internet Links

- Inverse Function Definition - Interactive Applet from AnalyzeMath
- Inverse Function - Interactive Tutorial from AnalyzeMath
- Inverse Functions Lesson - I from PurpleMath

(H) Homework

- Textbook - p. 122 # 16-18, 27-29, 37-39, 46-51