Lesson 7 - Algebraic Combinations of Functions

Math 2 Honors - Santowski

Fast Five

- Let's define f(x) = 2x – 3 and g(x) = x – 4
- (i) Evaluate f(0) + g(0)
- (ii) Write an equation for f(x) + g(x) and then evaluate (f + g)(0)
- (iii) Is their an observation that you notice/Is their a conjecture that you can make?
- (iv) HOW would you further validate your "conjecture"?

Fast Five

- Let's define f(x) = 2x – 3 and g(x) = x – 4
- (i) Evaluate f(0) x g(0)
- (ii) Write an equation for f(x) x g(x) and then evaluate (f x g)(0)
- (iii) Is their an observation that you notice/Is their a conjecture that you can make?
- (iv) HOW would you further validate your "conjecture"?

Lesson Objectives

- Perform a variety of operations with functions including evaluating functions; add, subtract, multiply, and divide; and invert functions and analyze in terms of domains and ranges
- Compose functions

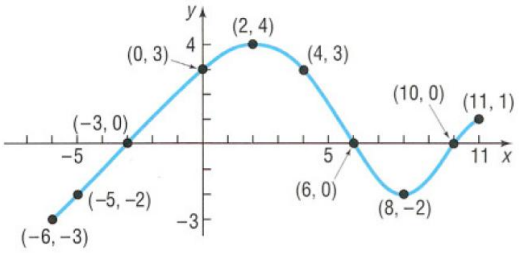
The **BIG** Picture

- And we are studying this because ....?
- The topics within the Math 2 Honors course will revolve around functions
- Functions will be a unifying theme throughout the course
- So a solid understanding of **what** functions are and **why** they are used and **how** they are used will be very important!

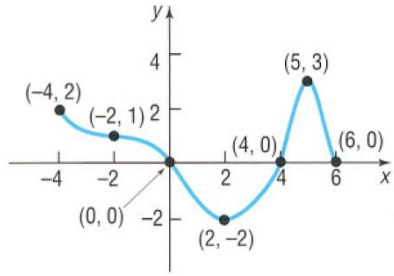
(A) Operations with Functions – Graphic Representation

- Given the functions on the following slides, evaluate:
- (a) f(6), g(-2), f(-1), g(4)
- (b) (f + g)(0), f + g (2), f + g (5), f + g (8)
- (c) (f – g)(-2), f – g (1), g – f (1), g – f (-5)
- (d) (fg)(2), fg (-2), gf (2), gf (-2)
- (e) (f/g) (1), f/g (4), g/f (2)
- (f) (1/f)(8), (1/f)(11), (1/g)(-4), (1/g)(-4), (1/g)(5)
- (g) fog(6), fog(-2), gof(6), gof(4)

Graphic Representation - Function f(x)

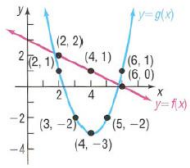


Graphic Representation - Function g(x)



Practice

The graph of two functions, f and g, is illustrated. Use the graph to answer parts (a)–(f).



(a) $(f + g)(2)$ (b) $(f + g)(4)$
(c) $(f - g)(6)$ (d) $(g - f)(6)$
(e) $(f \cdot g)(2)$ (f) $\left(\frac{f}{g}\right)(4)$

(B) Operations with Functions – Algebraic Representation

- Graph $f(x) = 1 - x^2$ on the TI-84
- Graph $g(x) = \frac{1}{2}x - 4$ on the TI-84
- Graph (f/g)(x) on the TI-84 and explain what happens at x = 8 and why.
- If g(x) where defined as g(x) = ax + b, what is the domain of (f/g)(x)
- What if $g(x) = ax^2 - c$?

(B) Operations with Functions – Algebraic Representation

- Given the functions on the previous slides, evaluate:
- (a) f(6), g(-2), f(-1), g(4)
- (b) (f + g)(0), f + g (2), f + g (5), f + g (8)
- (c) (f – g)(-2), f – g (1), g – f (1), g – f (-5)
- (d) (fg)(2), fg (-2), gf (2), gf (-2)
- (e) (f/g) (1), f/g (4), g/f (2)
- (f) (1/f)(8), (1/f)(11), (1/g)(-4), (1/g)(-4), (1/g)(5)
- (g) fog(6), fog(-2), gof(6), gof(4)

(C) Operations with Functions – Function Analysis

- Graph the following functions on the TI-84 or graphing software. Write an equation for each new function. Discuss the domain, range, and intercepts of the newly formed functions
- (i) f(x) + g(x) = (f + g)(x)
- (ii) f(x) - g(x) = (f – g)(x)
- (iii) g(x) - f(x) = (g – f)(x)
- (iv) | f(x) – g(x)| (absolute value)

(C) Operations with Functions – Function Analysis

- Graph the following functions on the TI-84 or graphing software. Write an equation for each new function. Discuss the domain, range, and intercepts of the newly formed functions
- (i) (fg)(x)
- (ii) (gf)(x)
- (iii) (f÷g)(x) = (f/g)(x)
- (v) (g÷f)(x) = (g/f)(x)

(D) Operations with Functions - Reciprocals

- Graph the following functions on the TI-84 or graphing software. Write an equation for each new function. Discuss the domain, range, asymptotes, and intercepts of the newly formed functions
- (i) 1/f(x)
- (ii) 1/g(x)

(E) Operations with Functions - Function Composition

- Now, instead of substituting another NUMBER into an equation, what happens if we substitute another FUNCTION into a function?
- Ex: let f(x) = | x | – 4 and let g(x) = x + 2
- ➔ Write an equation for f(g(x)) ➔ graph and comment?
- ➔ Write an equation for g(f(x)) ➔ graph and comment?
- Ex: let f(x) = 1 - x^2 and let g(x) = 2x + 6
- ➔ Write an equation for f(g(x)) ➔ graph and comment?
- ➔ Write an equation for g(f(x)) ➔ graph and comment?

(E) Operations with Functions - Function Composition

- So we have a way of creating a new function ➔ we can **compose** two functions which is basically a **substitution of one function into another**.
- we have a notation that communicates this idea ➔ if ***f*(*x*)** is one functions and ***g*(*x*)** is a second function, then the composition notation is ➔ **f o g (x)**

(F) WHY Composition of Functions – An Example

- ex 1. Andrew earns a daily wage of $20/h plus $15/d for travel expenses.
- We can write his daily earnings as an equation as Earnings = 20h + 15. However, Andrew also pays union fees at 2.5% of his daily earnings, which we can write as the equation Fees = 0.025 x (daily earnings)
- We can demonstrate with a table of values

Hours worked per day	Daily Earnings	Union Fees Paid
2		
5		

(F) WHY Composition of Functions – An Example

- What we see is that the one function value (daily earnings or E) is being substituted into the second function (Fees = 0.025 x daily earnings) in order to generate the value for the union fees.
- We can generate a direct formula for the union fees by substituting the earnings function into the Fees function as follows: Fees = 0.025(20h + 15).
- Hence, the Fees function is called a composed function as Fees(daily earnings) = 0.025 x daily earnings

(G) Composition of Functions – Example

- We will now define *f* and *g* as follows:
- *f* = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}

- We will now work with the composition of these two functions:
  - (i) We will evaluate *fog*(3) (or f(g(3))
  - (ii) evaluate *fog* (1)
  - (iii) evaluate *fog* (5) and see what happens ➔ why?
  - (iv) evaluate *gof* (9) and *g*(*f*(7) and *gog* (1)

(G) Composition of Functions – Example

- We will now define *f* and *g* as follows:
- *f* = {(3,2), (5,1), (7,4), (9,3), (11,5)}
- g = {(1,3), (2,5), (3,7), (4,9), (5,10)}

- We will now compose these two functions as follows:

- (i) we will substitute *g* into *f* which we will notate in two ways: *f*(*g*(*x*)) or as *fog*(*x*).
- We will evaluate *fog*(3) (or f(g(3)) ➔ we will substitute a specific *g* function value into *f*, that of *g*(3).
- The *g* function value at *x* = 3 is 7 (i.e. g(3) = 7)
- So now we evaluate f at the new value of x = 7 ➔we look at our *f* data and see that *f*(7) = 4

(G) Composition of Functions – Example

- We can define *f* and *g* differently, this time as formulas:

- $f(x) = x^2 - 3$ and $g(x) = 2x + 7$

- We will try the following:

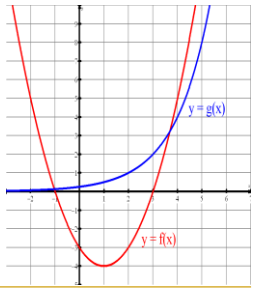
- (i) *f*(*g*(3)) or *fog*(3)
- (ii) *gof*(3) or *g*(*f*(3))
- (ii) *fog*(x) and *gof*(x)
- (ii) evaluate *fog* (5)
- (iii) evaluate *gof* (9) and *g*(*f*(7)) and *gog* (1)

(G) Composition of Functions – Example

- We can define *f* and *g* differently, this time as graphs:

- We will try the following:

- (i) *f*(*g*(3)) or *fog*(3)
- (ii) *gof*(3) or *g*(*f*(3))
- (iii) evaluate *fog*(2) and *fog*(-1)
- (iv) evaluate *gof*(0) and *g*(*f*(1)) and *gog*(2)



(G) Composition of Functions – Example

- We can define *f* and *g* differently, this time as formulas:

- Repeat if f(x) = $-2|x^2 + x - 1|$ and g(x) = 2^x

- We will try the following:

- (i) *f*(*g*(3)) or *fog*(3)
- (ii) *gof*(3) or *g*(*f*(3))
- (ii) *fog*(x) and *gof*(x)
- (ii) evaluate *fog* (5)
- (iii) evaluate *gof* (9) and *g*(*f*(7)) and *gog* (1)

(H) Internet Links

- College Algebra Tutorial Operations with Functions

- Polynomial Functions from AnalyzeMath

(I) Homework

- p. 115 # 15,17,23,25-29,37-45 odds,53-59 odds
- Sullivan, Sec 3.1, p219, Q52,57,68,69