Lesson 63 - Combinations

Math 2 Honors - Santowski

Intro Example

- List all 3 letter PERMUTATIONS of the letters a,b,c,d
- If the order of the letters DID NOT MATTER, which permutations you have listed are simply repetitions of the same arrangement?
- How many UNIQUE arrangements are you now left with?

Lesson Objectves

- (1) Use combinations to solve a counting problem involving groups.
- (2) Distinguish between problems involving permutations or combinations to count

Intro Example - Solution

- Since the order does not matter in combinations, there are clearly fewer combinations than permutations.
- The combinations are contained among the permutations -- they are a "subset" of the permutations.
- Each of those four combinations, in fact, will give rise to 3! Permutations ➔ P(3,3):

abc	abd	acd	bcd
acb	adb	adc	bdc
bac	bad	cad	cbd
bca	bda	cda	cdb
cab	dab	dac	dbc
cba	dba	dca	dcb

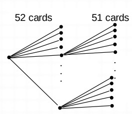
- So final answer ➔ 4 unique arrangements

Intro Example - Solution

- Since the order does not matter in combinations, there are clearly fewer combinations than permutations.
- The combinations are contained among the permutations -- they are a "subset" of the permutations.
- Each of those four combinations, in fact, will give rise to 3! Permutations ➔ P(3,3):
- So final answer ➔ 4 unique arrangements from the 24 arrangements that we started with
- Could we see a "general trend" that might lead to an algebraic shortcut (i.e. a formula?)

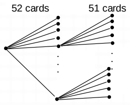
Intro Example

How many two-card hands can I draw from a deck when order does <u>not</u> matter (e.g., ace of spades followed by ten of clubs is <u>the same</u> as ten of clubs followed by ace of spades)



Intro Example

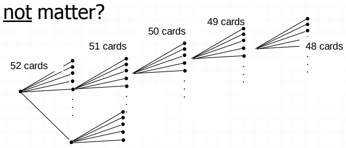
How many two-card hands can I draw from a deck when order does not matter (e.g., ace of spades followed by ten of clubs is the same as ten of clubs followed by ace of spades)



$$\frac{52x51}{2} = \frac{52!}{(52-2)!2}$$

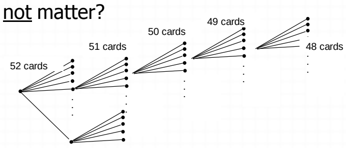
Intro Example

How many five-card hands can I draw from a deck when order does not matter?



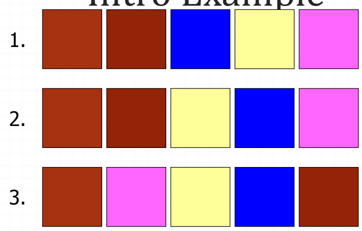
Intro Example

How many five-card hands can I draw from a deck when order does not matter?



$$\frac{52x51x50x49x48}{?}$$

Intro Example



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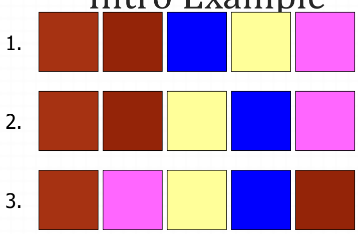
How many repeats total??

Intro Example



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i.e., how many different ways can you arrange 5 cards…?

Intro Example

That's a permutation without replacement.

5! = 120

$$\text{total\# of 5-card hands} = \frac{52x51x50x49x48}{5!} = \frac{52!}{(52-5)!5!}$$

Intro Example

- How many unique:
- 2-card sets out of 52 cards?
- 5-card sets?
- r-card sets?
- r-card sets out of n-cards?

Intro Example

- How many unique:
- 2-card sets out of 52 cards? $\frac{52x51}{2} = \frac{52!}{(52-2)!2!}$
- 5-card sets? $\frac{52x51x50x49x48}{5!} = \frac{52!}{(52-5)!5!}$
- r-card sets? $\frac{52!}{(52-r)!r!}$
- r-card sets out of n-cards? $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Overview

- When you need to count the number of groupings, without regard to order, then **combinations** are the way to go.
- Recall that **permutations** specifically count the number of ways a task can be arranged or ordered.
- That is the difference between the two, **permutations is with regard to order** and **combinations is without regard to order**.

Combination: Definition & Formula

- Combination:
- An arrangement of ***r*** objects, WITHOUT regard to ORDER and without repetition, selected from ***n*** distinct objects is called a combination of ***n*** objects taken ***r*** at a time.

$${}_nC_r = C(n,r) = \frac{n!}{(n-r)!r!}$$

- The number of such combinations is denoted by

Example A

- A teacher has 15 students and 5 are to be chosen to give demonstrations. How many different ways can the teacher choose the demonstrators given the following conditions:
- 1a. The order of the demonstrators is important?
- 1b. The order of the demonstrators is not important?

Example A – Solution 1a

- Keeping in mind that **order is important**, would this be a permutation or a combination? ➔ **permutation**
- **First we need to find *n* and *r* :**
- If *n* is the number of students we have to choose from, what do you think *n* is in this problem? ➔ There are 15 students.
- If *r* is the number of students chosen at a time, what do you think *r* is? ➔ 5 students are chosen to give demonstrations.
- So P(15,5) = 15!/10! = 15x14x13x12x11= 360360

Example A – Solution 1b

- Keeping in mind that **order is NOT important**, would this be a permutation or a combination? ➔ **combination** problem
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- **First we need to find *n* and *r* :**
- If *n* is the number of students we have to choose from, what do you think *n* is in this problem? ➔ There are 15 students to chose from.
- If *r* is the number of students chosen at a time, what do you think *r* is? ➔ 5 students are chosen to give demonstrations.
- So C(15,5) = P(15,5)/P(5,5) = 15!/(10!5!) = 3003

Example 2

- You are going to draw 4 cards from a standard deck of 52 cards. How many different 4 card hands are possible?
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Example 2 - Solution

- This would be a **combination problem**, because a hand would be a group of cards **without regard to order**. Note that if we were putting these cards in any kind of order, then we would need to use permutations to solve the problem.
- But in this case, **order does not matter**, so we are going to use **combinations**.
- **First we need to find *n* and *r* :**
- If *n* is the number of cards we have to choose from, what do you think *n* is in this problem? ➔ There are 52 cards in a deck of cards.
- If *r* is the number of cards we are using at a time, what do you think *r* is? ➔ We want 4 card hands.
- So C(52,4) = (52x51x50x49)/(4x3x2x1) = 270,725

Example 3 - Solution

- 3 marbles are drawn at random from a bag containing 3 red and 5 white marbles.
- **3a. How many different draws are there?**
- **3b. How many different draws would contain only red marbles?**
- **3c. How many different draws would contain 1 red and 2 white marbles?**
- **3d. How many different draws would contain exactly 2 red marbles?**
- http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut57_comb.htm#prob2a

PRACTICE PROBLEMS:

- Find the value of problems 1 through 6 and solve problems 7, 8,
- 1. ${}_6C_2$
  2. ${}_6C_4$
  3. ${}_{15}C_5$
  4. ${}_7C_7$
  5. $\frac{{}_6C_3 + {}_7C_3}{{}_{13}C_6}$
  6. $\frac{{}_7C_3 \cdot {}_6C_3}{{}_{14}C_4}$
- 7. We want to paint three rooms in a house, each a different color, and we may choose from seven different colors of paint. How many color combinations are possible for the three rooms?
- 8. If 20 boys go out for the football team, how many different teams may be formed, one at a time?
- 9. Two girls and their dates go to the drive-in, and each wants a different flavored ice cream cone. The drive-in has 24 flavors of ice cream. How many combinations of flavors may be chosen among the four of them if each one selects one flavor?

PRACTICE PROBLEMS - Answers

- Answers

1. 15
2. 15
3. 3,003
4. 1
5. $\frac{5}{156}$
6. $\frac{100}{143}$
7. 35
8. 167,960
9. 10,626

Summary of Counting Methods

Counting methods for computing probabilities

- **Permutations—order matters!**
  - **With replacement:** n^r
  - **Without replacement:** $n(n-1)(n-2)\ldots(n-r+1) = \frac{n!}{(n-r)!}$
- **Combinations—Order doesn't matter**
  - **Without replacement:** $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

HOMEWORK

- S10.3, p647, Q1-8,16-19,24,25,28,31,33,34-38,42,50