Lesson 5 –Solving Absolute Value Equations & Inequalities

Math 2 Honors - Santowski

Lesson Objectives

- Solve and graph absolute value equations
- Solve and graph absolute value inequalities
- Reinforce the equality of multiple representations of solving methods

BIG Picture

- One constant theme in our course will be studying various ways that numbers are INTERRELATED and the first model used to study these interrelationships will be LINEAR MODELS. Our BASE linear function is y = x
- A BASE function can be “modified” to create “new” functions. The **absolute value function** is a “modified” linear function.
- With new functions, we can **expand our applications** of a “base” function
- And as in previous lessons → **HOW** do we work with these models **ALGEBRAICALLY, GRAPHICALLY & NUMERICALLY**??

(A) Absolute Value – The Concept Geometrically

- There are several ways to understand the concept of absolute value:
  - (a) physics
  - (b) numbers
- So we can evaluate this as | -3 | = ??? and | +3 | = ??

(A) Absolute Value – The Concept Geometrically

- There are several ways to understand the concept of absolute value:
  - (i) the idea of an "undirected" distance, speed, force ...etc.... (think scalar quantities in Physics)
  - (ii) number line - the points +3 and -3 are both the same distance from 0. Hence, they have the same absolute value (think: distance from 0 (or from a rest position))
- So we can evaluate this as | -3 | = 3 and | +3 | = 3

(A) Absolute Value – The Concept Graphically

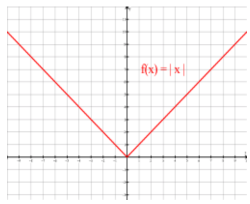
- Create a table of values for the function y = | x |
- Graph y = |x| from your table of values

(A) Absolute Value – The Concept Graphically

- Our graph will start with a table of values

x	y
-10.00000	10.00000
-9.00000	9.00000
-8.00000	8.00000
-7.00000	7.00000
-6.00000	6.00000
-5.00000	5.00000
-4.00000	4.00000
-3.00000	3.00000
-2.00000	2.00000
-1.00000	1.00000
0.00000	0.00000
1.00000	1.00000
2.00000	2.00000
3.00000	3.00000
4.00000	4.00000
5.00000	5.00000
6.00000	6.00000
7.00000	7.00000
8.00000	8.00000
9.00000	9.00000
10.00000	10.0000

- Here is a graph of y = |x|



(A) Absolute Value – The Concept Algebraically

- Absolute value can be seen as a function defined by the following piecewise function:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

- So what does this mean:

$$|-3| = -(-3) = 3$$
$$|3| = (3) = 3$$

(A) Absolute Value – The Concept Graphically

- We can look at a graph of y = |x| to solidify some of the geometric & algebraic ideas:
- Notice the graph of the function y = x and how part of this graph (that part which is on the left side of x = 0) seems to have been reflected over the y-axis
- (Hence the idea of if x < 0, then |x| = -x)

(A) Absolute Value – The Concept

- So then the meaning behind these ideas is that absolute value seems to be a “mathematical operation” which takes either a positive or negative value as its “input” and returns a positive value as its “output”

(B) Working With Absolute Value: Examples

- Simplify the following
  - (i) | -8 |
  - (ii) |0 – 6 |
- Simplify
  - (i) | 5 – 2 |
  - (ii) | 2 – 5 |
- evaluate |-2|, |4|, and 8|-5|
- evaluate |3| + |-5|
- evaluate -|-3| + 2|3| - 6|-5| - |-6| + 2|0|

(C) Equations with Absolute Value – Geometric (Number line)

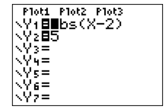
- We can consider a simple example of |*x*| = 5 in terms of a number line (geometrically)➔ I wish to find the number(s) on the number line that are 5 units from 0 (or from a “center”)
- So we get two possibilities, one case in which *x* is -5 (5 units to the left of 0) or the other case in which *x* is +5 (5 units to the right of 0)
- Therefore, I can re-express the original equation as two “new” equations:
- (i) – (*x*) = -(-5) = 5 and
- (ii) + (*x*) = 5,
- which then gives me the solutions of 5 or -5 as my values for *x*

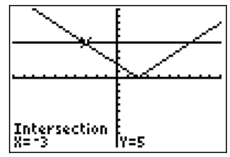
(C) Equations with Absolute Value – Geometric (Number line)

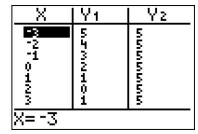
- So now solve $|x – 2| = 5$ geometrically
- We can visualize the process on a number line again ➔ I am looking for 2 values that are both 5 units from "0" (or from the center);
- however, the values have been translated right by 2 units (Recall the transformational significance of (x-2)) ➔ so my original values of ± 5 being translated right by 2 gives me $x = 7$ or $x = -3$
- We can also solve this equation graphically:

(C) Equations with Absolute Value – Graphic Solution

- So here are the graphs of:
- y = | x – 2 |
- y = 5







(C) Equations with Absolute Value

- Reconsider the example of $|x – 2| = 5$ algebraically now
- So then the expression $(x – 2)$ could represent a positive "input" ➔ in which case (x – 2) = 5
- Or our expression $(x – 2)$ could represent a negative "input" ➔ in which case $-(x – 2) = 5$
- So we have 2 equations for solving the equations:
- (i) $+(x – 2) = 5$ and
- (ii) $–(x – 2) = 5$
- The two equations, then, give us the answers x = 7 or x = -3 as expected

(C) Equations with Absolute Value

- Solve and verify $|2x - 3|- 4 = 3$
- Solve and verify $-2|x/2 + 3| - 4 = -10$
- Solve and verify $|2x - 2| = x + 1$
- Solve and verify $|x + 1| = x – 3$
- Solve and verify $|x^2 – 4x – 5| = 7$
- Solve each of the equations GRAPHICALLY as well

(C) Equations with 2 Absolute Values

- But what about the equation $|2x – 1|=|4x + 3|$

(C) Equations with 2 Absolute Values

- Solve and verify $|2x – 1|=|4x + 3|$
- So here we have to consider various possibilities:
- The "input" of (2x – 1) could be +(2x – 1) or $–(2x – 1)$
- Likewise, the input of (4x + 3) could be +(4x + 3) or $–(4x + 3)$
- So then we could potentially set up 4 equations:
- +(2x – 1) = +(4x + 3)
- -(2x – 1) = -(4x + 3)
- +(2x – 1) = -(4x + 3)
- -(2x – 1) = +(4x + 3)
- But note that really equations #1 and #2 are the same, as are equations #3, #4 ➔ so I only really need 2 of these equations, say #1 and #3
- So now solve this equation (x = -2, -1/3) (Solution at the following website

(D) Inequalities with Absolute Value

- Let's work through some algebraic solutions to inequalities with absolute value and simultaneously consider the graphic solutions.
- In each case, we will record our final solution in set and interval notation

(D) Inequalities with Absolute Value

- Solve and verify:
- $| 2x - 4 | < 10$
- $| 9x - 2 | \leq 1$
- $| 2x - 3 | > 7$
- $-| 7t + 10 | \geq 3$
- $| 3x + 2 | < 0$
- $| 4x + 15 | < -2$

- Solve and verify:
- $| 2x - 4 | < x - 2$
- $|2x - 3 | \geq -1/2x + 3$
- $| x^2 - 4 | > x + 2$

Internet links

- http://www.youtube.com/watch?v=rC4bRSQetvQ
- http://tutorial.math.lamar.edu/Classes/Alg/SolveAbsValueIneq.aspx
- http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut21_abseq.htm
- http://www.analyzemath.com/Equations/Absolute_Value_Tutorial.html
- http://www.purplemath.com/modules/solveabs1.htm

(H) Homework

- p. 69 # 33-39 odds, 65
- p. 67 # 19-24, 53-61