Permutations

S10.2

Lesson Objectives

- Use the Fundamental Counting Principle to determine the number of outcomes in a problem.

- Use the idea of permutations to count the number of possible outcomes in a problem.

Fundamental Counting Theorem

- It will allow us to count the number of ways a task can occur given a series of events

- The Fundamental Counting Principle is the guiding rule for finding the number of ways to accomplish a series of tasks.

Fundamental Counting Theorem

- This principle states that *"If there are **r** ways to do one thing, and **s** ways to do another, and **t** ways to do a third thing, and so on ..., then the number of ways of doing all those things at once is* ***r*** *x* ***s*** *x* ***t*** *x ..."*

EXAMPLE 1

- You are taking a test that has five True/False questions. If you answer each question with True or False and leave none of them blank, in how many ways can you answer the whole test?

EXAMPLE 1

- **Let's use the basic counting principle**:
- There are 5 stages or events: question 1, question 2, question 3, question 4, and question 5.
- There are **2 choices for each question.**

- Putting that all together we get:
- **Q 1 Q2 Q3 Q4 Q5**
- **# of ways to answer test ➔** 2 x 2 x 2 x 2 x 2 = **32**
- **So there are 32 different ways to answer the whole test.**

EXAMPLE 1

- Now, determine the probability of:

- (A) all correct answers are TRUE
- (B) three correct answers are FASLE
- (C) At least 2 answers are TRUE

EXAMPLE 2

- A deli has a lunch special which consists of a sandwich, soup, dessert and drink for \$4.99. They offer the following choices:
- **Sandwich**: chicken salad, ham, and tuna, and roast beef
- **Soup**: tomato, chicken noodle, vegetable
- **Dessert**: cookie and pie
- **Drink**: tea, coffee, coke, diet coke and sprite

- How many lunch specials are possible?

EXAMPLE 2

- **Let's use the basic counting principle**: There are 4 stages or events: choosing a sandwich, choosing a soup, choosing a dessert and choosing a drink.

- There are **4 choices for the sandwich, 3 choices for the soup, 2 choices for the dessert and 5 choices for the drink.**
- Putting that all together we get:

- **Sand. Soup Dessert Drink ➔ # of lunch specials**
- 4 x 3 x 2 x 5 = 120

- **So there are 120 lunch specials possible**

EXAMPLE 2

- Now to incorporate probabilities ....

- (A) Determine the probability that the meal contains a ham sandwich
- (B) Determine the probability that the meal contains a ham sandwich AND a coke
- (C) Determine the probability that the meal contains a ham sandwich OR a coke

EXAMPLE 3

- A company places a 6-symbol code on each unit of product. The code consists of 4 digits, the first of which is the number 5, followed by 2 letters, the first of which is NOT a vowel.

- How many different codes are possible?

EXAMPLE 3

- **Let's use the basic counting principle**:
- There are 6 stages or events: digit 1, digit 2, digit 3, digit 4, letter 1, and letter 2.
- In general there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The first digit is limited to being the number 5, so there is only one possibility for that one. There are no restriction on digits 2 – 4, so each one of those has 10 possibilities.
- In general, there are 26 letters in the alphabet. The first letter, cannot be a vowel (a, e, i, o, u), so that means there are 21 possible letters that could go there. The second letter has no restriction, so there are 26 possibilities for that one.
- Putting that all together we get:

- **digit 1 digit 2 digit 3 digit 4 letter 1 letter 2**
- **# of codes 1 x 10 x 10 x 10 x 21 x 26 = 546000**
- **So there are 546000 different 6-symbol codes possible.**

EXAMPLE 3

- To incorporate probabilities
- (A) Determine the probability of having the last letter being a vowel?
- (B) Determine the probability of having a letter duplicated?

Permutations

- A permutation is an arrangement of objects, without repetition, and order being important.
- Since a permutation is the number of ways you can arrange objects, it will always be a whole number.
- The ***n*** value is the total number of objects to chose from. The ***r*** is the number of objects your actually using.
- The two key things to notice about permutations are that there is **no repetition of objects allowed** and **that order is important**.

Permutations

- Permutations specifically count the number of ways a task can be arranged or ordered
- Notations & Formula → $_nP_r = P(n,r) = \frac{n!}{(n-r)!}$
- Where n! means n x (n – 1) x (n – 2) x .... x 3 x 2 x 1

Factorials

- Find 7!
- Find (7!)/(4!)
- Find (7!)/(5!2!)

Permutations

- Example 1: How many permutations of the letters ABCD are there?

Permutations

- Example 1: List all permutations of the letters ABCD

ABCD ABDC ACBD ACDB ADBC ADCB	BACD BADC BCAD BCDA BDAC BDCA	CABD CADB CBAD CBDA CDAB CDBA	DABC DACB DBAC DBCA DCAB DCBA

- Now, if you didn't actually need a listing of all the permutations, you could use the formula for the number of permutations.
- There are 4 objects and you're taking 4 at a time.
- ${}_4P_4 = 4! / (4-4)! = 4! / 0! = 24 / 1 = 24.$

EXAMPLE 1

- In how many ways can a class of 20 members select a president, vice president and treasury, assuming that the same person cannot hold more than one office.

EXAMPLE 1

- **Since we are choosing offices, which is a way to rank members, that means order is important. So we can use permutations to help us out here.**

- **First we need to find *n* and *r* :**

- *n* is the number of members we have to choose from ➔ There are 20 members in this problem.

- *r* is the number of members we are selecting for offices at a time ➔ There are 3 offices.
- **Putting this into the permutation formula we get:**

$$_{20}P_3 = P(20,3) = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20\times19\times18\times17!}{17!} = 6840$$

EXAMPLE 2

- How many different arrangements can be made using two of the letters of the word TEXAS if no letter is to be used more than once?

EXAMPLE 2

- **Since we are arranging letters, this means order is important. So we can use permutations to help us out here.**
- **First we need to find *n* and *r* :**
- *n* is the number of letters we have to choose from ➔ There are 5 letters in TEXAS.
- *r* is the number of letters we are using at a time ➔ We are using 2 letters at a time.
- **Putting this into the permutation formula we get:**

$$_5P_2 = P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5\times 4\times 3!}{3!} = 20$$

EXAMPLE 2

- How many different arrangements can be made using two of the letters of the word TEXAS if no letter is to be used more than once?

- And to incorporate probabilities:

- Determine the probability that the “word” contains a T

- Determine the probability that the “word” contains a T or an X

Permutations with Identical Objects

- Ex 1 → What are the arrangements of the letters a, a, b ?
- Ex 2 → In how many ways can the letters of the word "coffee" be arranged?

Permutations with Identical Objects

- So we need to adjust our formula as →
- Number of distinguishable permutations that can be formed from a collection of n objects where the first object appears k1 times, the second object k2 times, and so on is:
- $n! / (k1! \cdot k2! \cdots kt!)$
- where $k1 + k2 + \ldots + kt = n$

Permutations with Identical Objects

- How many distinguishable words can be formed from the letters of JEFF?

Permutations with Identical Objects

- How many distinguishable words can be formed from the letters of JEFF?
- Solution: $n = 4, kj = 1, ke = 1, kf = 2$
- $n!/(kj! \cdot ke! \cdot kf!) = 4!/(1!\ 1!\ 2!) = 12$
- List:
- JEFF, JFEF, JFFE, EJFF, EFJF, EFFJ,
- FJEF, FEJF, FJFE, FEFJ, FFJE, and FFEJ

Permutations with Identical Objects

- How many distinguishable words can be formed from the letters of MISSISSIPPI?

Permutations with Identical Objects

- How many distinguishable words can be formed from the letters of MISSISSIPPI?
- Solution:
- $n = 11$, $km = 1$, $ki = 4$, $ks = 4$, $kp = 2$
- $n!/(km! \cdot ki! \cdot ks! \cdot kp!) = 11!/(1!\ 4!\ 4!\ 2!) = 34{,}650$

Permutations with Circular Arrangements

- Ex ➔ In how many ways can 4 people be arranged in a line?

- Ex ➔ In how many ways can the same 4 people now be seated around a round table?

Example

- In how many ways can 6 people be seated at a round table?

Example – Solution

- How many ways are there to sit 6 people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table?

- First, place the first person in the north-most chair
  - Only one possibility
- Then place the other 5 people
  - There are P(5,5) = 5! = 120 ways to do that
- By the product rule, we get 1*120 =120

- Alternative means to answer this:
- There are P(6,6)=720 ways to seat the 6 people around the table
- For each seating, there are 6 "rotations" of the seating
- Thus, the final answer is 720/6 = 120

EXAMPLE

- A five digit integer is formed from the digits 1,2,3,4,5,6,7,8,9 with no digit used more than once.

- (a) How many integers can be formed?
- (b) Determine the probability that the integer begins with a 3
- (c) Determine the probability that the integer contains a 6.

EXAMPLE

- In how many ways can 9 different books be arranged on the shelf so that:
- a) 3 of the books are always together?
- b) 3 of the books are never together?

Homework

- S10.1; p633; Q8,26-29,38,39,44
- S10.2; p639; Q1,3,4, 9,13,17,21,25,27,29,35,37,39,43,47,48,49,52