Lesson 43 – Sine Law & The Ambiguous Case

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Lesson Objectives

- Understand from a geometric perspective WHY the ambiguous case exists

- Understand how to identify algebraically that their will be 2 solutions to a given sine law question

- Solve the 2 triangles in the ambiguous case

- See that the sine ratio of a acute angle is equivalent to the sine ratio of its supplement

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(A) Warm Up Problems

- Determine the measure of an angle whose sine ratio is 0.75
- Solve the equation $\sin(x) = 0.75$ for x
- Solve the equation $x = \sin^{-1}(0.75)$

- What is the difference in meaning amongst these 3 questions??

- Solve the following equations for x:
- $x = \sin^{-1}(0.89)$ $x = \cos^{-1}(.11)$
- $x = \sin^{-1}(0.25)$ $x = \tan^{-1}(3.25)$
- $\sin(x) = 0.45$ $\sin(x) = 0.6787$

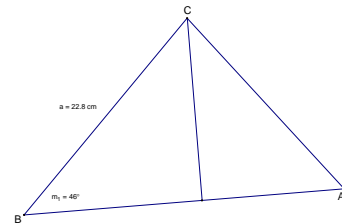- Explain why it is IMPOSSIBLE to solve $\sin^{-1}(1.25) = x$

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(A) Warm Up Problems

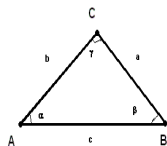- Explain how to find the height (altitude) of this non-right triangle

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(B) Sine Law – Scenario #1

- Let's work through 2 scenarios of solving for $\angle B$:

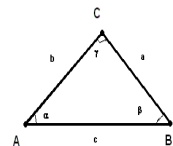- Let $\angle A = 30°$, $a = 3$ and $b = 2$ (so the longer of the two given sides is opposite the given angle)

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(B) Sine Law – Scenario #1

- Let's work through 2 scenarios of solving for $\angle B$:

- Let $\angle A = 30°$, $a = 3$ and $b = 2$ (so the longer of the two given sides is opposite the given angle)

- Then $\sin \beta = b \sin \alpha / a$
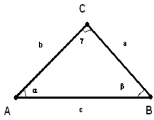- And $\sin \beta = 2 \sin 30 / 3$
- So $\angle B = 19.5°$

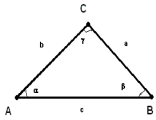2/22/2011   Math 2 Honors - Santowski   6

(C) Sine Law – Scenario #2

- In our second look, let's change the measures of a and b, so that a = 2 and b = 3 (so now the shorter of the two given sides is opposite the given angle)
- ■

(C) Sine Law – Scenario #2

- In our second look, let's change the measures of a and b, so that a = 2 and b = 3 (so now the shorter of the two given sides is opposite the given angle)
- ■

- Then $\sin \beta = b \sin \alpha / a$
- And $\sin \beta = 3 \sin 30 / 2$
- So $\angle B = 48.6°$
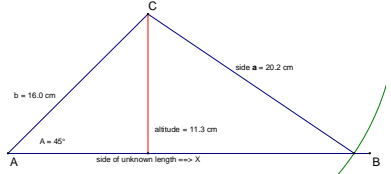
- BUT!!!!! …….. there is a minor problem here …..

(F) Considerations with Sine Law

- If you are given information about non-right triangle and you know 2 angles and 1 side, then ONLY one triangle is possible and we never worry in these cases

- If you know 2 sides and 1 angle, then we have to consider this "ambiguous" case issue

  - If the side opposite the given angle IS THE LARGER of the 2 sides ➔ NO WORRIES
  - **If the side opposite the given angle IS THE SHORTER of the 2 sides ➔ ONLY NOW WILL WE CONSIDER THIS "ambiguous" case**

- WHY????

Case #1 – if a > b
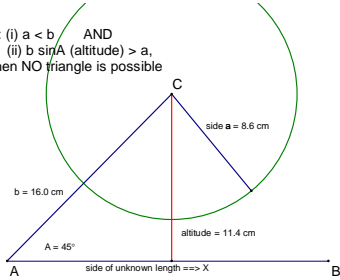
If: (i) a > b, then ONE OBTUSE triangle is possible

Case #2 - if a < b
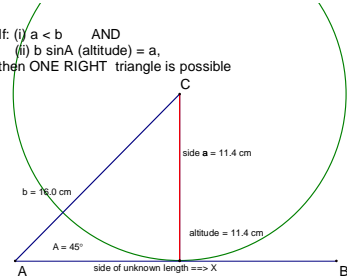
If: (i) a < b AND
(ii) b sinA (altitude) > a,
then NO triangle is possible

Case #3 – if a < b

If: (i) a < b AND
(ii) b sinA (altitude) = a,
then ONE RIGHT triangle is possible

Case #4 – the Ambiguous Case

If: (i) $a < b$, AND
   (ii) if $b\sin A < a$
then TWO triangles are possible

$\sin(m\angle CB'B) = 0.83$
$m\angle CB'B = 55.72°$

$a = 10.2$ cm

$b = 14.4$ cm

$m\angle B'BC = 55.72°$
$\sin(m\angle B'BC) = 0.83$

$m\angle CBA = 124.28°$
$\sin(m\angle CBA) = 0.83$

altitude = 8.5 cm

$m\angle B'BC + m\angle CBA = 180.00°$

side of unknown length ==> X   B'   B

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Case #4 – the Ambiguous Case

If: (i) $a < b$, AND
   (ii) if $b\sin A < a$
then TWO triangles are possible

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$m\angle CB'B = 55.72°$

$a = 10.2$ cm

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$m\angle CBA = 124.28°$
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altitude = 8.5 cm

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side of unknown length ==> X   B'   B

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Case #4 – the Ambiguous Case

If: (i) $a < b$, AND
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then TWO triangles are possible

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$a = 10.2$ cm

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altitude = 8.5 cm

$m\angle B'BC + m\angle CBA = 180.00°$

side of unknown length ==> X   B'   B

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Summary

- Case 1 ➔ if we are given 2 angles and one side ➔ proceed using sine law **(ASA)**
- Case 2 ➔ if we are given 1 angle and 2 sides and the side opposite the given angle is LONGER ➔ proceed using sine law

- if we are given 1 angle and 2 sides and the side opposite the given angle is SHORTER ➔ proceed with the following "check list"

- Case 3 ➔ if the product of "$b\sin A > a$", NO triangle possible
- Case 4 ➔ if the product of "$b\sin A = a$", ONE triangle
- Case 5 ➔ if the product of "$b\sin A < a$" TWO triangles

- RECALL that "$b\sin A$" represents the altitude of the triangle

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Summary

| $\angle A < 90°$ (acute)	Conditions	Number and Type of Triangles Possible
	$a < b\sin A$	no triangle
	$a = b\sin A$	one right triangle
	$b\sin A < a < b$	two triangles—one acute, one obtuse
	$a \geq b$	one triangle

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Examples of Sine Law

- if $\angle A = 44°$ and $\angle B = 65°$ and $b = 7.7$ find the missing information.

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Examples of Sine Law

- if ∠ A =44.3º and a=11.5 and b=7.7 find the missing information.

Examples of Sine Law

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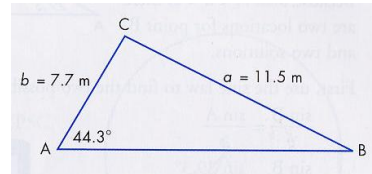
Examples of Sine Law

- if ∠ A=29.3º and a=12.8 and b = 20.5

Examples of Sine Law

- if ∠ A=29.3 and a=12.8 and b = 20.5
- All the other cases fail, because bsinA<a<b 10<a (12.8)<20.5, which is true.
- Then we have two triangles, solve for both angles
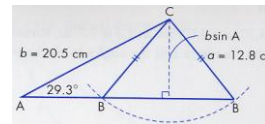
Examples of Sine Law

- Solve triangle *PQR* in which ∠ *P* = 63.5° and ∠ *Q* = 51.2° and *r* = 6.3 cm.

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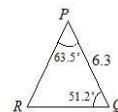
Examples of Sine Law

- ex. 1. In ΔABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles

- ex. 2. Solve ΔABC if ∠ A = 37.7°, a = 30 cm, b = 42 cm

Examples of Sine Law

- ex. 1. In Δ ABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- First test ➔side opposite the given angle is longer, so no need to consider the ambiguous case ➔ i.e. a > b ➔ therefore only one solution

- ex. 2. Solve Δ ABC if ∠ A = 37.7, a = 30 cm, b = 42 cm
- First test ➔ side opposite the given angle is shorter, so we need to consider the possibility of the "ambiguous case" ➔ a < b ➔ so there are either 0,1,2 possibilities.
- So second test is a calculation ➔ Here a (30) > b sin A (25.66), so there are two cases

Homework

- HW

- Nelson Questions - OPTIONAL but good practice ➔ (link attached in HW section of website): any of Q5,6,8