Lesson 3 – Linear Equations

Math 2 Honors Santowski

Fast Five – Warm up & Challenge

- Given a parallelogram whose vertices are defined by the co-ordinates A(-1,4), B(2,-2), C(6,3), and D(x,y). Determine the co-ordinates of point D
- Determine the co-ordinates of the intersection point of the diagonals.
- Determine which point(s) are equidistant from K(-2,2) and M(3,6)

Lesson Objectives

- Write a linear equation in two variables given sufficient information
- Introduce the term **linear function**
- Express a linear equation in a variety of forms including slope-intercept form, standard form, and point-slope form
- Write an equation for a line that contains a given point and is parallel or perpendicular to a given line

BIG Picture

- Since we have defined Math as a study of numbers, we also note that part of our definition of math focuses on the INTERRELATIONSHIPS that exist with our numbers.
- One constant theme in our course will be studying various ways that numbers are INTERRELATED and the first model used to study these interrelationships will be LINEAR MODELS.
- Recognize that **different forms** of linear equations **communicate different key information**

BIG Picture

- In studying the relationship between the height of a ramp and the distance a marble will travel when rolled off the ramp, the following data was generated by my Grade 9 class:

Height (cm)	2.3	3.2	4.2	5.6	7.2	10	12.7	14
Distance (cm)	53	60	70	76	93	131	143	150

- From this data, we can put together a variety of REPRESENTATIONS to help us analyze and understand the data:
  - (1) Equation:
  - (2) Graph:
  - (3) Number Analysis:

(A) Slope Calculation

- To calculate the slope between any 2 points, (x_1,y_1) and (x_2,y_2), we can use the "formula"

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

- This formula "works" as long as .....???

(B) Linear Equations

- We can determine the equations of linear equations if:
- (a) we know 2 points that the line passes through
  - Ex. A(-3,5) & B(3,7)
- (b) if we know the slope of the line and a point through which the line passes
  - Ex. If slope = -1/2 and P(-3,6)

(B) Linear Equations - Modeling

- If shares in Microsoft where $25.50/share on June 11 and are $22.74/share on August 17, determine:
- (a) the slope of the linear equation that can be used to model the share price of Microsoft shares.
- (b) Interpret the MEANING of the slope.
- (c) If I set the "y-intercept" to be New Years, Jan 1, 2009, determine the equation of the line which models the share price of Microsoft

(C) Slope Interpretation – Rate of Change

- An ABSOLUTELY vital thing to understand about slope is that the slope of the segment between any 2 points represents the AVERAGE RATE OF CHANGE between those 2 points
- Ex. Determine the average rate of change between A(1,1) and B(4,9)

(D) Forms of Linear Equations

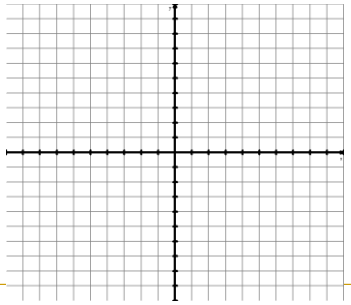
- Linear equations can be written in many forms:
- (A) Slope-intercept form ➔ ex. In the linear equation y = 4x – 5, the slope of the line is 4 while the y-intercept is at (0,-5)
- So the general form looks like y = mx + b where m is the slope and b is the y-intercept

(D1) Slope-Intercept form of Linear Equations

- On a grid, sketch the lines defined by the following linear equations:
- (a) y = 2x + 3
- (b) y = -1/2x + 2
- (c) y = 1-x
- (d) y = 3
- (e) x = 2
- Determine the equation of the lines defined by the following conditions:
  - (a) passing through (1,2) and (4,-4)
  - (b) having a slope of -¼ and passing through (6,2)

(D1) Slope-Intercept form of Linear Equations

-



(D) Forms of Linear Equations

- Linear equations can be written in many forms:
- (B) Point - Slope ➔ ex. If we know that a line of slope 3 passes through the point (1,2), we can quickly write the linear equation as: ????

(D2) Point-Slope Form of Linear Equations

- If a line passes through P(1,2) and has a slope of 3, then

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$3 = \frac{y-2}{x-1}$$

$$y - 2 = 3(x-1)$$

- So in general for a line of slope m passing through the point (h,k), the eqn becomes:
- y – k = m(x – h)

- Write in point-slope form, the equations o f the lines:
- (a) slope of 8 passing through (-3,6)
- (b) passing through (2,-3) and (-1,9)

(D) Forms of Linear Equations

- Linear equations can be written in many forms:
- (C) Standard Form ➔ ex. If we know that a line of slope 3 passes through the point (1,2) in the form of Ax + By + C = 0

(D) Forms of Linear Equations

- If a line passes through P(1,2) and has a slope of 3, then

$$Ax + By + C = 0$$

$$3x - 1y + C = 0$$

$$3(1) - 1(2) + C = 0$$

$$3 - 2 + C = 0$$

$$C = -1$$

$$\therefore 3x - y - 1 = 0$$

- So in general our equation has the form Ax + By + C = 0

- Write in standard form, the equations o f the lines:
- (a) slope of 3/2 passing through (-2,4)
- (b) passing through (1,3) and (-1,9)

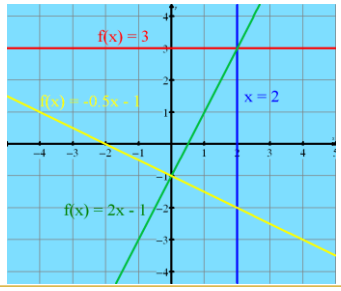
(E) Special Lines & Slopes

- There are 2 special lines that deserve attention:
- Horizontal lines have a slope of 0 and have an equation y = ***k***, where ***k*** represents any arbitrary y value that is constant for every ordered pair on that line
- Vertical lines have an undefined slope and have an equation in the form of x = ***h***, where ***h*** represents any arbitrary value that is constant for every ordered pair on that line

(E) Special Lines & Slopes

- There are 2 special cases of lines that deserve attention:
- Parallel lines are lines that have the same slope
- Perpendicular lines have slopes that are negative multiplicative inverses (i.e. Negative reciprocals of each other)

(E) Special Lines



(E) Special Lines & Slopes

- (i) Determine the equation of the line parallel to 2x – 3y + 5 = 0 and passing through the point (4,-7)
- (ii) Determine the equation of the perpendicular bisector of the line joining the points (2,3) and (4,7)

(F) Forms of Linear Equations

- Given the following equations, rearrange the equation and graph it on the TI-84
- What special observation do you notice?

$$\frac{x}{2}+\frac{y}{3}=1$$

$$\frac{x}{4}-\frac{y}{1}=1$$

$$\frac{y}{2}-\frac{x}{3}=1$$

(F1) – Intercept Form of Linear Equations

- So an equation written in the form of

$$\frac{x}{a}+\frac{y}{b}=1$$

or

$$bx+ay=ab$$

- tell us the x- and y-intercepts of the line (and even the slope can be easily calculated as ...?)

Homework

- p. 26 # 17-21, 27-37 odds, 47-51 odds, 60-61
- From the Sullivan Text, p284; Q15,17,21,25,31,33,35,37,53,54

(G) Extension

- Graph the parabola $y = x^2$ and highlight the points A(1,1) and B(2,4)
- (a) A ***secant*** line passes through 2 points on the curve and will go through the points A and B ➔ Determine the slope of this secant line
- (b) Now move point B to (1.5, 2.25) and find the secant slope
- (c) Move B to (1.1, 1.21) and find the secant slope
- (d) Move B to (1.01, 1.0201) and find the secant slope
- (e) Let B be located at (x+h, $(x+h)^2$). Find the slope of the secant AB
- (f) Explain the statement $\lim_{h\to 0}\frac{(x+h)^2-x^2}{(x+h)-x}$