Lesson 34 – Modeling with Exponential Functions

Math 2 Honors - Santowski

Comparing Data Models

- Data set #1

X	0	1	2	3	4	5	6
y	50	75	112.5	168.75	253.13	379.69	569.53

- Data set #2

X	0	1	2	3	4	5	6
y	5	7.75	10.5	13.25	16	18.75	21.5

- Data set #3

X	0	1	2	3	4	5	6
y	3	6	11	18	27	38	51

(A) Comparing Data Models

- (a) Graph the data on a scatter plot
- (b) How can you graphically analyze the data to help determine a model for the data?
- (c) How can you numerically analyze the data to help determine a model for the data?
- (d) Write an equation to model the data. Justify your choice of models.

(A) Modeling Example #1

- The following data table shows the relationship between the time (in hours after a rain storm in Manila) and the number of bacteria (#/mL of water) in water samples from the Pasig River:
- (a) Graph the data on a scatter plot
- (b) How can you graphically analyze the data to help determine a model for the data?
- (c) How can you numerically analyze the data to help determine a model for the data?
- (d) Write an equation to model the data. Justify your choice of models.

Time (hours)	# of Bacteria
0	100
1	196
2	395
3	806
4	1570
5	3154
6	6215
7	12600
8	25300

(A) Modeling Example #2

- The value of my car is depreciating over time. I bought the car new in 2002 and the value of my car (in thousands) has been tabulated below:

Year	2002	2003	2004	2005	2006	2007	2008	2009	2010
Value	40	36	32.4	29.2	26.2	23.6	21.3	19.1	17.2

- (a) Graph the data on a scatter plot
- (b) How can you graphically analyze the data to help determine a model for the data?
- (c) How can you numerically analyze the data to help determine a model for the data?
- (d) Write an equation to model the data. Justify your choice of models.

(A) Modeling Example #3

- The following data table shows the historic world population since 1950:

Year	1950	1960	1970	1980	1990	1995	2000	2005	2010
Pop (in millions)	2.56	3.04	3.71	4.45	5.29	5.780	6.09	6.47	6.85

- (a) Graph the data on a scatter plot
- (b) How can you graphically analyze the data to help determine a model for the data?
- (c) How can you numerically analyze the data to help determine a model for the data?
- (d) Write an equation to model the data. Justify your choice of models.

Exponential Models

- Key Data feature ➔ common RATIO between consecutive terms
- So that must mean our model must account for the RATIO between consecutive terms …..

(C) Exponential Modeling - Growth

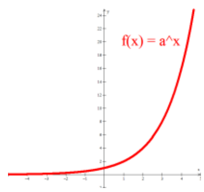
- In general, however, the algebraic model for exponential growth is $y = c(a)^x$ where *a* is referred to as the growth rate (provided that a > 1) and *c* is the initial amount present and x is the number of increases given the growth rate conditions.
- All exponential equations are also written in the form $y = c(1 + r)^x$ where c is a constant, r is a positive rate of change and 1 + r > 1, and x is the number of increases given the growth rate conditions.

(C) Exponential Modeling - Decay

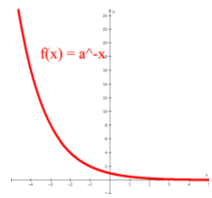
- In general, however, the algebraic model for exponential decay is $y = c(a)^x$ where *a* is referred to as the decay rate (provided that 0 < a < 1) and *c* is the initial amount present and x is the number of decreases given the decay rate conditions.
- All exponential equations are also written in the form $y = c(1 + r)^x$ where c is a constant, r is a negative rate of change and 1 + r < 1, and x is the number of increases given the decay rate conditions.

(F) Exponential Functions

- The features of the parent exponential function $y = a^x$ (where a > 1) are as follows:



- The features of the parent exponential function $y = a^x$ (where 0 < a < 1) are as follows:



(F) Exponential Functions

- The features of the parent exponential function $y = a^x$ (where a > 1) are as follows:
- Domain ➔
- Range ➔
- Intercept ➔
- Increase on ➔ Asymptote ➔
- As x →-∞, y →
- As x → ∞, y →

- The features of the parent exponential function $y = a^x$ (where 0 < a < 1) are as follows:
- Domain ➔
- Range ➔
- Intercept ➔
- Increase on ➔ Asymptote ➔
- As x →-∞, y →
- As x → ∞, y →

Working with Exponential Models

- Populations can also grow exponentially according to the formula $P = P_o(1.0125)^n$. If a population of 4,000,000 people grows according to this formula, determine:
- 1. the population after 5 years
- 2. the population after 12.25 years
- 3. when will the population be 6,500,000
- 4. what is the average annual rate of increase of the population

Working with Exponential Models

- The value of a car depreciates according to the exponential equation $V(t) = 25{,}000(0.8)^t$, where t is time measured in years since the car's purchase. Determine:

- 1. the car's value after 5 years
- 2. the car's value after 7.5 years
- 3. when will the car's value be $8,000
- 4. what is the average annual rate of decrease of the car's value?

Working with Exponential Models

- Bacteria grow exponentially according to the equation $N(t) = N_o 2^{(t/d)}$ where N(t) is the amount after a certain time period, N_o is the initial amount, t is the time and d is the doubling period

- A bacterial strain doubles every 30 minutes. If the starting population was 4,000, determine

- 1. the number of bacteria after 85 minutes
- 2. the number of bacteria after 5 hours
- 3. when will the number of bacteria be 30,000
- 4. what is the average hourly rate of increase of the # of bacteria?
- 4. what is the average daily rate of increase of the # of bacteria?

(C) Examples

- ex. 1 A bacterial strain doubles every 30 minutes. If there are 1,000 bacteria initially, how many are present after 6 hours?

- ex 2. The number of bacteria in a culture doubles every 2 hours. The population after 5 hours is 32,000. How many bacteria were there initially?

Working with Exponential Models

- Radioactive chemicals decay over time according to the formula $N(t) = N_o 2^{(-t/h)}$ where N(t) is the amount after a certain time period, N_o is the initial amount, t is the time and h is the halving time ➔ which we can rewrite as $N(t) = N_o(1/2)^{(t/h)}$ and the ratio of t/h is the number of halving periods.

- Ex 1. 320 mg of iodine-131 is stored in a lab but it half life is 60 days. Determine

- 1. the amount of I-131 left after 10 days
- 2. the amount of I-131 left after 180 days
- 3. when will the amount of I-131 left be 100 mg?
- 4. what is the average daily rate of decrease of I-131?
- 4. what is the average monthly rate of decrease of I-131?

(E) Half Life - Examples

- Ex 2. Health officials found traces of Radium F beneath P044. After 69 d, they noticed that a certain amount of the substance had decayed to $1/\sqrt{2}$ of its original mass. Determine the half-life of Radium F

(C) Exponential Modeling

- Investments grow exponentially as well according to the formula $A = P_o(1 + i)^n$. If you invest $500 into an investment paying 7% interest compounded annually, what would be the total value of the investment after 5 years?

- (i) You invest $5000 in a stock that grows at a rate of 12% per annum compounded quarterly. The value of the stock is given by the equation $V = 5000(1 + 0.12/4)^{4x}$, or $V = 5000(1.03)^{4x}$ where x is measured in years.
  - (a) Find the value of the stock in 6 years.
  - (b) Find when the stock value is $14,000

(C) Examples

- ex. 4 Populations can also grow exponentially according to the formula $P = P_o(1 + r)^n$. If a population of 4,000,000 people grows at an average annual rate of increase of 1.25 %, find population increase after 25 years.
- ex 5. The population of a small town was 35,000 in 1980 and in 1990, it was 57,010. Create an algebraic model for the towns population growth. Check your model using the fact that the population was 72800 in 1995. What will the population be in 2010?

(E) Examples

- ex 3. Three years ago there were 2500 fish in Loon Lake. Due to acid rain, there are now 1945 fish in the lake. Find the population 5 years from now, assuming exponential decay.
- ex 4. The value of a car depreciates by about 20% per year. Find the relative value of the car 6 years after it was purchased.
- Ex 5. When tap water is filtered through a layer of charcoal and other purifying agents, 30% of the impurities are removed. When the water is filtered through a second layer, again 30% of the remaining impurities are removed. How many layers are required to ensure that 97.5% of the impurities are removed from the tap water?

Homework