Lesson 32 – Solving Radical Equations

Math 2 Honors - Santowski

Lesson Objectives

- solutions to radical equations can be graphically or algebraically presented
- what do we mean by "equivalent" systems, why do they arise, and what do they mean, and why are they important
- incorporate the idea that root functions have restrictions

(A) Solving Radical Equations – Example #1

- We will investigate the idea of "equivalent systems"
- Use a graph to solve the equation $2\sqrt{x+5}=8$
- Use a graph to solve the equation $x+5=16$
- Explain what is meant by "equivalent systems" given your 2 solutions to the 2 equations

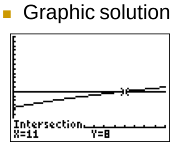
$$S_1=\begin{cases} f(x)=2\sqrt{x+5} \\ g(x)=8 \end{cases} \qquad S_2=\begin{cases} f(x)=x+5 \\ g(x)=16 \end{cases}$$

(A) Solving Radical Equations – Example #1

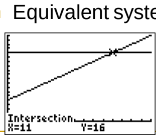
- Algebraically solve $2\sqrt{x+5}=8$

(A) Solving Radical Equations – Example #1

- Graphic solution is:



- Equivalent system:



- Algebra solution is:

$2\sqrt{x+5}=8 \quad \text{where } x\geq -5$

$\sqrt{x+5}=4$

$\left(\sqrt{x+5}\right)^2=(4)^2$

$x+5=16$

$x=11$

(B) Solving Radical Equations – Example #2

- We will investigate the idea of "equivalent systems"
- Use a graph to solve the equation $3+\sqrt{x+1}=2x$
- Use a graph to solve the equation $x+1=4x^2-12x+9$
- Is this an example of "equivalent systems" given your 2 solutions to the 2 equations?

(B) Solving Radical Equations – Example #2

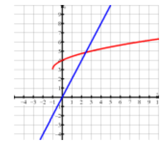
- Algebraically solve $3+\sqrt{x+1}=2x$

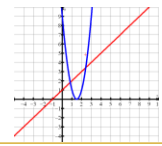
(B) Solving Radical Equations – Example #2

- And the algebraic solution

$$3+\sqrt{x+1}=2x \quad \text{where } x\geq -1$$
$$\sqrt{x+1}=2x-3$$
$$\left(\sqrt{x+1}\right)^2=(2x-3)^2$$
$$x+1=4x^2-12x+9$$
$$0=4x^2-13x+8$$
$$\therefore x=\frac{-(-13)\pm\sqrt{(-13)^2-4(4)(8)}}{2(4)}$$
$$\therefore x=0.83,\ 2.43$$

- Explain what the term "extraneous solution" means
- Explain WHY they occur.





(C) Solving Radical Equations – Example #3

- We will investigate the idea of "equivalent systems"
- Use a graph to solve the equation $-1+\sqrt{x}=\sqrt{2x+1}$
- Use a graph to solve the equation $4x=x^2$
- Is this an example of "equivalent systems" given your 2 solutions to the 2 equations?

(C) Solving Radical Equations – Example #3

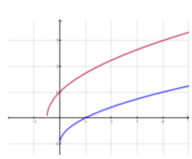
- Algebraically solve $-1+\sqrt{x}=\sqrt{2x+1}$

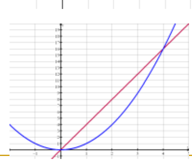
(C) Solving Radical Equations – Example #3

- And the algebraic solution

$$-1+\sqrt{x}=\sqrt{2x+1} \quad \text{where } x\geq 0$$
$$\left(-1+\sqrt{x}\right)^2=\left(\sqrt{2x+1}\right)^2$$
$$1-2\sqrt{x}+x=2x+1$$
$$-2\sqrt{x}=x$$
$$\left(-2\sqrt{x}\right)^2=(x)^2$$
$$4x=x^2$$
$$0=x^2-4x$$
$$0=x(x-4)$$
$$\therefore x=0,\ 4$$

- Explain what the term "extraneous solution" means
- Explain WHY they occur.





(D) Solving Radical Equations – Example #4

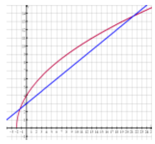
- Graphically solve $\sqrt{8x+16}=0.5x+3$
- Algebraically solve $\sqrt{8x+16}=0.5x+3$

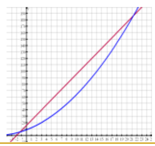
(D) Solving Radical Equations – Example #4

- And the algebraic solution

$$\sqrt{8x+16} = 0.5x+3 \quad \text{where } x \geq -2$$
$$\left(\sqrt{8x+16}\right)^2 = (0.5x+3)^2$$
$$8x+16 = 0.25x^2+3x+9$$
$$0 = 0.25x^2 - 5x - 7$$
$$\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(0.25)(-7)}}{2(0.25)}$$
$$\therefore x = -1.314,\ 21.314$$

- Explain what the term "extraneous solution" means
- Explain WHY they occur.





(E) Radical Equations – Predicting Extraneous Solutions

- Solve and verify (algebraically)

$$\sqrt{x+7} = x+1$$

- Q? In what domain do you expect the solution to be?????

(E) Radical Equations – Predicting Extraneous Solutions

- Solve and verify (algebraically)

$$\sqrt{x+7} = x+1$$
$$\left(\sqrt{x+7}\right)^2 = (x+1)^2$$
$$x+7 = x^2+2x+1$$
$$0 = x^2+2x+1-(x+7)$$
$$0 = x^2+x-6$$
$$0 = (x+3)(x-2)$$
$$\therefore x = -3 \quad \text{and} \quad x = 2$$

Let's verify our x values:

$$x = 2$$
$$\sqrt{2+7} = 2+1$$
$$3 = 3$$

$$x = -3$$
$$\sqrt{-3+7} = -3+1$$
$$2 = -2$$

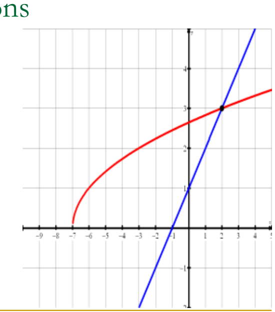
So x = -3 doesn't verify

So $x = 2$

(E) Radical Equations – Predicting Extraneous Solutions

- Let's graphically solve

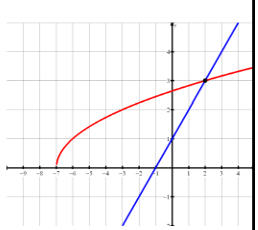
$$\sqrt{x+7} = x+1$$



(E) Radical Equations – Predicting Extraneous Solutions

- Looking CAREFULLY at our graphic solution ➔ Is there not another logical way to eliminate the "extraneous" solution ???

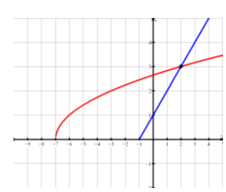
$$\sqrt{x+7} = x+1$$



(E) Radical Equations – Predicting Extraneous Solutions

- HINT: Range of this radical function is ➔ y > 0
- Which should imply that f(x) = x+1 should be considered for only output values of y > 0
- So when is x + 1 > 0 ➔ for x > -1
- So our considerations for solutions should take into account x > -1 as well as x > -7
- Recall that the algebra gave us x = -3 and x = 2 ➔ so it should be obvious that x = -3 is an extraneous solution

$$\sqrt{x+7} = x+1$$



(F) Further Examples

- Solve and verify without a calculator:

$$\text{(a) } \sqrt{3x+4} = \sqrt{x}+2$$
$$\text{(b) } \sqrt{x-1} - \sqrt{x+1} = -1$$

- Are the following statements (a) always true, (b) sometimes true or (c) never true

$$\text{(a) } 2\sqrt[4]{3x} = \sqrt[4]{3x+15}$$
$$\text{(b) } \sqrt{x-6} = 3+\sqrt{x}$$

(F) Further Examples

- Solve algebraically and verify

$$\text{(a) } 1+\frac{\sqrt{y+4}}{\sqrt{y-3}} = \frac{7}{\sqrt{y-3}}$$
$$\text{(b) } \sqrt[3]{2x+1} = 3$$
$$\text{(c) } \sqrt[3]{1-3x} - 4 = 0$$

Homework

- p. 542 # 13-23 odds, 24, 39, 41, 43, 53, 57, 61-63