

Lesson 28 – Solving Rational Inequalities

Math 2 Honors

1

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Fast Five

▶ Solve $\frac{x}{2x-1} \leq 1$

▶ 2

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Lesson Objectives

- ▶ 1. Review multiple methods for solving inequalities
- ▶ 2. Review the “sign chart” as an algebraic “tool” to help solve rational inequalities
- ▶ 3. Solve rational inequalities

▶ 3

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(A) Solving Rational Inequalities -

- ▶ We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$
- ▶ (1) Interpret the MEANING of the inequality
- ▶ (2) Set up a table of values and answer the question from the data table
- ▶ (3) Prepare a graph BY HAND and answer the question
- ▶ (4) Graph on the calculator
- ▶ (5) Solve using algebra (GASP!!!!)

▶ 4

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(A) Solving Rational Inequalities - ANS

- ▶ We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$
- ▶ (1) Interpret the MEANING of the inequality → when is the line $g(x)=2x-3$ higher than the rational function

$$f(x) = \frac{x+2}{x-4}$$

▶ 5

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(A) Solving Rational Inequalities - ANS

- ▶ We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$
 - ▶ (2) Set up a table of values and answer the question from the data table
- | x | y ₁ | y ₂ |
|----|----------------|----------------|
| -2 | 0 | -7 |
| -1 | -0.2 | -5 |
| 0 | -0.5 | -3 |
| 1 | -1.0 | -1 |
| 2 | -2.0 | 1 |
| 3 | -5.0 | 3 |
| 4 | undef | 5 |
| 5 | 7.0 | 7 |
| 6 | 4.0 | 9 |
| 7 | 3.0 | 11 |
| 8 | 2.5 | 13 |
| 9 | 2.2 | 15 |
| 10 | 2.0 | 17 |

▶ 6

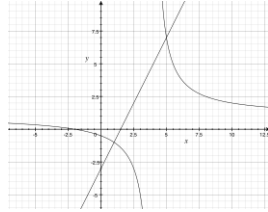
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(A) Solving Rational Inequalities - ANS

- We will work with the rational inequality

$$\frac{x+2}{x-4} < 2x-3$$

- (3) Prepare a graph BY HAND and answer the question



- Recall long division (or synthetic in this case)

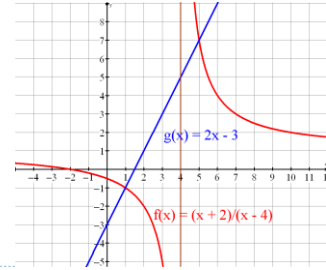
$$\frac{x+2}{x-4} = 1 + \frac{6}{x-4}$$

7

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(A) Solving Rational Inequalities - ANS

- We will graph using technology $\frac{x+2}{x-4} < 2x-3$



8

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(A) Solving Rational Inequalities - ANS

- We will solve using algebra
- Now that we have done all the algebra and factored everything, use a sign chart to solve the inequality:

$$\begin{aligned} \frac{x+2}{x-4} < 2x-3; x \neq 4 \\ \frac{x+2}{x-4} < \frac{2x-3}{1} \\ \left(\frac{x+2}{x-4}\right)\left(\frac{1}{1}\right) < \left(\frac{x-4}{x-4}\right)\left(\frac{2x-3}{1}\right) \\ 0 < \left(\frac{x-4}{x-4}\right)\left(\frac{2x-3}{1}\right) - \left(\frac{x+2}{x-4}\right)\left(\frac{1}{1}\right) \\ 0 < \frac{2x^2-11x+12}{x-4} - \frac{x+2}{x-4} \\ 0 < \frac{2x^2-12x+10}{x-4} \\ 0 < \frac{2(x-5)(x-1)}{x-4} \end{aligned}$$

	$(-\infty, 1)$	$(1, 4)$	$(4, 5)$	$(5, \infty)$
$(x-5)$	-ve	-ve	-ve	+ve
$(x-1)$	-ve	+ve	+ve	+ve
$(x-4)$	-ve	-ve	+ve	+ve
R(x)	-ve	+ve	-ve	+ve

9

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(A) Solving Rational Inequalities - ANS

- We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$
- So regardless of how we set it up, we come up with the same solution $\rightarrow \{x \in \mathbb{R} \mid 1 < x < 4 \text{ or } x > 5\}$

10

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(B) Further Examples

- To illustrate and emphasize the algebraic methods (and as a way of incorporating past algebra skills), solve the following rational inequalities:

(i) $\frac{x}{2x-1} \leq 1$

(ii) $\frac{x-2}{2(x-3)} > \frac{x}{x+3}$

11

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(B) Further Examples - Solutions

- Here is the algebra
- Here is the sign chart:

(i) $\frac{x}{2x-1} \leq 1$
 $\left(\frac{x}{2x-1}\right)\left(\frac{1}{1}\right) \leq \left(\frac{1}{1}\right)\left(\frac{2x-1}{2x-1}\right); x \neq \frac{1}{2}$
 $\frac{x}{2x-1} \leq \frac{2x-1}{2x-1}$
 $0 \leq \frac{2x-1}{2x-1} - \frac{x}{2x-1}$
 $0 \leq \frac{x-1}{2x-1}; x \neq \frac{1}{2}$

	$x < \frac{1}{2}$	$(\frac{1}{2}, 1)$	$x > 1$
$x-1$	-ve	-ve	+ve
$2x-1$	-ve	+ve	+ve
R(x)	+ve	-ve	+ve

- So the solution is that $x > 1$ or $x < \frac{1}{2}$

12

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(B) Further Examples - Solutions

▶ Here is the algebra

$$\frac{x-2}{2(x-3)} > \frac{x}{x+3}$$

$$\frac{(x-2)(x+3)}{2(x-3)(x+3)} > \frac{2x(x-3)}{2(x-3)(x+3)}$$

$$0 > \frac{2x^2 - 6x}{2(x-3)(x+3)} - \frac{x^2 + x - 6}{2(x-3)(x+3)}$$

$$0 > \frac{x^2 - 7x + 6}{2(x-3)(x+3)}$$

$$0 > \frac{(x-6)(x-1)}{2(x-3)(x+3)}$$

▶ Here is the sign chart:

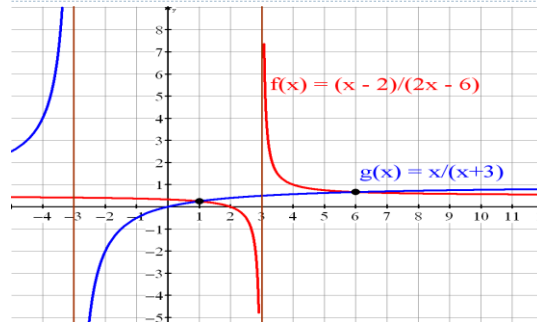
	$x < -3$	$(-3, 1)$	$(1, 3)$	$(3, 6)$	$x > 6$
$x-6$	-ve	-ve	-ve	-ve	+ve
$x-1$	-ve	-ve	+ve	+ve	+ve
$x-3$	-ve	-ve	-ve	+ve	+ve
$x+3$	-ve	+ve	+ve	+ve	+ve
$R(x)$	+ve	-ve	+ve	-ve	+ve

▶ So the solution is that $x > 1$ or $x < \frac{1}{2}$

▶ 13

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(B) Further Examples - Solutions



▶ 14

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Homework

▶ p. 518 # 29-41 odd, 43-46, 51-53, 55

▶ 15

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Calculus EXTENSION

▶ You will work with the equation $r(x) = \frac{x^2+1}{x^2-x-2}$

▶ Perform a long division (will synthetic not work??) and interpret the quotient

▶ Evaluate and interpret $\lim_{x \rightarrow \infty} \frac{x^2+1}{x^2-x-2}$ ▶ Solve and interpret $\frac{x^2+1}{x^2-x-2} = 1$

▶ 16

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Calculus EXTENSION - Answers

▶ You will work with the equation $r(x) = \frac{x^2+1}{x^2-x-2}$ ▶ Perform a long division (will synthetic not work??) and interpret the quotient $r(x) = \frac{x^2+1}{x^2-x-2} = 1 + \frac{x+3}{x^2-x-2}$ ▶ Evaluate and interpret $\lim_{x \rightarrow \infty} \frac{x^2+1}{x^2-x-2} = \lim_{x \rightarrow \infty} \left(1 + \frac{x+3}{x^2-x-2} \right) = 1$ ▶ Solve and interpret which is where the rational function crosses its horizontal asymptote!! $1 + \frac{x+3}{x^2-x-2} = 1$
 $\therefore \frac{x+3}{x^2-x-2} = 0$

$$\therefore x = -3$$

▶ 17

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